

A Macrofinance View of U.S. Sovereign CDS Premiums*

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Abstract

The premiums on U.S. sovereign credit default swaps (CDS) have risen to persistently elevated levels since the financial crisis. We examine whether these premiums reflect the probability of a fiscal default – a state in which the budget balance can no longer be restored by raising taxes or eroding the real value of debt by increasing inflation. We develop an equilibrium macrofinance model in which the fiscal and monetary policy stances jointly endogenously determine the nominal debt, taxes, inflation, and growth. We show how the CDS premiums reflect the endogenous risk-adjusted probabilities of fiscal default. The calibrated model is consistent with elevated levels of CDS premiums but leaves dynamic implications quantitatively unresolved.

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1 Introduction

The 2008 credit crisis brought about significant changes in the markets for sovereign credit default swaps (CDS) of economically developed countries. The near zero trading volumes at near zero premiums observed in late 2007 expanded to active trading at substantial premiums of hundreds of basis points. Although the crisis eventually subsided, the premiums on sovereign CDS remain elevated and are nowhere close to the pre-crisis levels. In this paper, we seek to determine the risks that are so richly compensated in these markets.

Evidently, the answer seems obvious given that CDS are designed to insure against default. However, at the height of the crisis, in the United States five-year protection cost 100 bps, and it has traded at around 20 bps since 2014. Is a U.S. default that likely or does the severity of the expected losses justify such premiums? Basic reasoning suggests that the answer to both these questions would be no. For instance, some observers believe that the U.S. will never default because it can either “inflate away” its debt obligations or increase taxes, or both.

Our objective in this paper is to establish a quantitative benchmark for the compensation based on default risk. Thus, we make a first step toward developing a formal macro-based framework that will allow us to evaluate the likelihood of a sovereign default and the associated risk premium. The advantage of such an approach is that it allows us to study the effects of monetary and fiscal policies.

There are many good reasons to suppose there could be other determinants of the CDS premiums. For instance, the friction arising from various institutional features of the CDS markets, such as the margin requirements, counterparty risk, capital

constraints, and credit event determination, could be responsible for part of the observed premium. We do not disagree with such a view, and leave these explanations for future research.

Because this is the first step in addressing these questions, we keep our setting as simple as possible. We directly specify the dynamics of a number of key variables, such as the aggregate output, consumption growth, and government expenditure. The glue that holds these variables together and allows us to investigate the questions of interest is the government budget constraint (GBC). Because the government can tax aggregate output and issue new nominal debt to finance its expenditure and repay its outstanding debt, the GBC endogenously determines the relation between the issued debt and taxes.

We specify monetary policy via a Taylor rule that determines the behavior of inflation. In an endowment economy, monetary policy usually does not have real effects. In contrast, in our setting with the GBC featuring nominal debt, inflation has real quantitative effects. For example, fiscal policy responds to the amount of outstanding debt and expected growth in the economy.

Our model endogenously allows for states of the economy in which the budget balance can no longer be restored by raising taxes or eroding the real value of debt by creating inflation. In such situations, the government will have no choice other than to default on its debt. We refer to such a scenario as a *fiscal default*. Episodes of fiscal stress arise in our model because we assume that an increase in the tax rate has a small, negative effect on future long-term output growth. Thus, attempts to achieve a balanced budget by raising taxes may reduce the taxable income, which will further exacerbate the fiscal conditions. Fiscal default then arises when taxes

cannot be raised further without reducing the future tax revenues, in the spirit of a Laffer curve. This trade-off prompts our specification of a maximum amount of debt outstanding, which is related to the expenditure and tax rates, and ultimately determines the timing of the default.

We complement our model with a representative agent who has [Epstein and Zin \(1989\)](#) preferences and uses her marginal rate of substitution to value assets. Consumption features a time-varying conditional mean similar to [Bansal and Yaron \(2004\)](#). These assumptions allow us to value nominal defaultable securities using inflation and the timing of default implied by the GBC and policy rules.

Qualitatively, we find that the model provides significant insights into the macroeconomic determinants of the CDS premiums on U.S. Treasury debt. In the model, periods of high government debt endogenously correspond to investors' high marginal utility. As government expenditure rises, the government edges closer to the fiscal limit at which point further tax increases will reduce tax income. The default probabilities and the likelihood of incurring losses on government debt thus increase during periods of high marginal utility. Thus, insurance companies underwriting government debt face missed required payments during periods of high marginal utility. To compensate for their exposure to this risk, the insurance companies charge high risk premiums. Although the average losses on government debt are small and governments tend to make small average payments to insurers, the defaults occur in the worst of all states. In the context of our model, risk premiums thus make up a substantial part of the CDS premiums beyond the expected losses.

We use our model to explore a rare and severe endogenous event that may affect the U.S. economy. Although the default is severe, exogenous consumption and therefore

the marginal rate of substitution are not affected. In this sense, we are contemplating a mechanism that is similar to that of [Barro \(2006\)](#), although without the rare disasters in consumption. One implication of this setting is that the derived CDS premiums are likely to be conservative.

Quantitatively, we find that our model can generate periods of persistently elevated CDS premiums, similar to the recent U.S. experience. In the simulations, our model produces CDS premiums of up to 100 bps on an annual basis, which is similar to the peak values of the U.S. CDS premiums around the time of the financial crisis in 2008. Perhaps more importantly, however, our model predicts periods of persistently elevated CDS premiums even during calmer times because in our setup with recursive preferences, investors anticipate and dislike occasional shocks to default probabilities, which results in elevated CDS premiums. The model is thus consistent with the notion that CDS premiums reflect investors' rational forecasts of the likelihood of U.S. fiscal stress.

We use the model to revisit the idea of avoiding default by increasing taxes or inflating the government debt away. We represent these notions by changing the respective fiscal and monetary policy stances. Raising less debt or responding to inflation less aggressively leads to a decline in the average probability of default and increased CDS premiums. This happens because changes in the government's respective policy stances also increase the volatility of taxes and inflation, thus implying higher risk premiums. We also evaluate changes in the debt duration, which serve as a metaphor for the combination of the Federal Reserve's quantitative easing and the U.S. Treasury's debt maturity extension programs. Our model implies that a shorter duration leads to an increase in CDS premiums due to the rollover risk.

Because we intentionally use off-the-shelf modeling elements, the resulting model contains a number of simplifications. Some of the simplifications are evident, such as the exogenous dependence of expected output on taxes and government policies that do not change at default. However, others suggest model misspecifications that are shared with a large body of the asset-pricing literature. The most notable features are the downward-sloping real yield curve, constant price of inflation risk, counter cyclicity of inflation that contradicts recent evidence. We comment more on these in the relevant parts of the manuscript.

Further, the model leaves quantitatively unresolved the dynamic relation between the key exogenous variables, such as consumption growth and government expenditures, and the endogenous variables, which are inflation, debt-to-GDP, interest rates, and CDS premiums. In the case of consumption growth, the signs of cross-correlations in the model are consistent with those in the data, but are difficult to measure precisely. In the case of government expenditures, the cross-correlations are well-measured but the model overshoots the ones in the data.

Our work adds a macrofinance perspective to the growing body of literature on sovereign default and the pricing of sovereign default risk. Although there is considerable interest in sovereign default in the macro and finance literature, these fields have evolved somewhat separately. Our paper is a first step toward synthesizing insights from the macro and finance literature and distilling them into a quantitative framework that relies on standard building blocks.

A number of papers in the finance literature are based on the contingent claims approach, which was originally developed to analyze defaultable corporate debt. In this approach, a bond is treated as a (short put) option on the value of a firm's

unlevered assets, and default is triggered by a combination of a firm's difficulty in servicing debt and the provision of bankruptcy laws. When applying the contingent claims approach to sovereign debt, unlevered assets are replaced with the present value of future output. The key difficulty is that there is no bankruptcy law at the sovereign level, so the cause and timing of default are not clear.

Strategic defaults occur when penalties such as limited access to international debt markets and trade sanctions are outweighed by the debt burden. In the contingent claims framework, these considerations lead to default when the present value of output under default exceeds the present value under continuation of debt service (Kulatilaka and Marcus, 1987). Gibson and Sundaresan (2005) endogenize the strategic default trigger and the resulting risk premiums (credit spreads) by embedding a bargaining game between the sovereign and the creditors. The issue with this approach is that there is inconclusive empirical evidence regarding the impact of penalties on sovereign defaults. In our model, the government defaults when it runs out of available debt-servicing tools (issuing new debt, inflating debt, raising taxes), and thus can no longer meet its long-term financial obligations.

The affine models of sovereign default focus on estimating realistic models of the default probabilities and default risk premiums in emerging economies using an intensity-based approach. Duffie, Pedersen, and Singleton (2003) estimate a model of Russian credit spreads. Pan and Singleton (2008) estimate the risk-adjusted default arrival rate and loss given default using sovereign CDS. Ang and Longstaff (2013) estimate a joint affine model of U.S. CDS, U.S. states, and Eurozone sovereigns. We use our model to provide the economic underpinning of defaults and to distinguish between the risk-adjusted and actual probabilities of default (recovery is fixed at a constant for simplicity, but this can be easily extended).

[Augustin and Tedongap \(2016\)](#) also value Eurozone CDS from the perspective of an Epstein-Zin agent. The key difference from our approach is that they also follow an intensity-based approach, that is, they assume a function connecting a sovereign's default probability to the expected consumption growth and macro volatility. In our model, the default probability is determined endogenously via the interaction between fiscal policy and the GBC combined with monetary policy.

[Bhamra, Kuehn, and Strebulaev \(2010\)](#), [Chen \(2010\)](#), and [Chen, Collin-Dufresne, and Goldstein \(2009\)](#) value corporate bonds by linking models of endogenous corporate default with habit-based and recursive preferences. The valuation mechanism in our paper is similar in that a high default risk premium generates substantial credit spreads while keeping the default probabilities realistically low. However, in contrast to this line of work, we focus on sovereign default, which entails a different default trigger. [Borri and Verdelhan \(2012\)](#) use a risk-sensitive consumption-based model based on habit preferences to study sovereign default premiums in emerging markets.

Similar to the contingent claims framework, strategic default is at the core of the international macroeconomics literature on sovereign default, in the spirit of [Eaton and Gersovitz \(1981\)](#). Recent studies along these lines include [Arellano \(2008\)](#), [Arellano and Ramanarayanan \(2012\)](#), and [Yue \(2010\)](#). This important line of work solves general equilibrium endowment models of small open economies in which governments default strategically in the best interest of households and analyzes the implications for sovereign credit spreads. Our paper differs from these studies along several dimensions. From a quantitative viewpoint, we use a risk-sensitive framework in which risk premiums make up a sizeable component of the spreads. Furthermore, we emphasize the limitations of using fiscal instruments to restore the budget balance when

in default.

In the latter respect, our paper is closer to work of [Leeper \(2013\)](#) on fiscal uncertainty and debt limits. [Bi and Leeper \(2013\)](#) and [Bi and Traum \(2012\)](#) analyze business cycle models that explicitly allow for fiscal limits and apply them to the recent period of heightened sovereign risk in Greece. In contrast to our paper, they do not focus on CDS premiums or spreads, and do not use a risk-sensitive framework. Moreover, we emphasize a growth channel of fiscal policy via elevated tax rates that depress the future growth prospects, which is absent in their work. This channel emerges endogenously from the recent work linking long-run risks with fiscal policy in models of endogenous growth ([Croce, Kung, Nguyen, and Schmid, 2012](#); [Croce, Nguyen, and Schmid, 2013](#)), and is consistent with the empirical evidence, as documented in [Easterly and Rebelo \(1993\)](#) and [Mendoza and Tesar \(1998\)](#). In this respect, our work is closest to [Chen and Verdelhan \(2015\)](#), who examine the links between taxation and sovereign risk, but do not focus on U.S. CDS premiums.

2 A Primer on U.S. Sovereign CDS

We start by providing a basic background on corporate CDS. This information motivates our interest in sovereign CDS and we use it to explain the important differences between the two types of contracts.

2.1 Corporate CDS

Prior to the introduction of the Big and Small Bang protocols in 2009, a long position in a corporate CDS contract required no payments upfront. Premiums were paid

quarterly. In case of a credit event, the contract was settled via a delivery of allowed bonds of the corporate entity or a cash payment with the amount determined in a CDS auction in exchange for the full par (notional) paid in cash.

The Big and Small Bang protocols have codified the use of bond auctions to determine the payments of the long party. The auctions take place within 30 days after a credit event and allow the delivery of any bonds of a defaulted company from a pre-specified list leading to the cheapest-to-deliver option. The value of this option should be small for corporate entities because their bonds tend to trade at approximately the same price after a credit event ([Chernov, Gorbenko, and Makarov, 2013](#)).

The protocols also established standardized CDS premiums (100 bps for investment grade and 500 bps for speculative grade entities). The standardized CDS premiums simplified the netting and offsetting of positions but introduced the need to pay an upfront fee to ensure that the present values of all the cash flows line up. The CDS contracts continue to be quoted on a par basis (zero payment upfront). For this reason, we ignore these institutional details in this paper.

The quarterly premiums can be easily estimated using the replication argument applied to par bonds. Par bonds have coupon payments such that the bond value is equal to par immediately after a coupon payment. Consider buying a corporate bond and eliminating the credit risk by buying protection via a CDS contract. The running payment is the difference between the corporate yield and the CDS premium. By no-arbitrage, this difference should be equal to the risk-free rate. Absent the liquidity premium and credit risk in the U.S. Treasury, the yield on a Treasury par bond of matching maturity would be a good proxy for this payment.

In practice, par bonds may not be available, so it may be difficult to find bonds with

matching maturity, corporate bonds may be much more expensive to short due to their scarcity, or the Treasury yields may reflect a non-negligible liquidity premium. These complications introduce the non-zero difference between the CDS premium and a bond's credit spread, known as the CDS-bond basis (Blanco, Brennan, and Marsh, 2005; Longstaff, Mithal, and Neis, 2005). Typically, the basis is positive and reflects the cost of shorting a corporate bond. Because these costs vary with the trading party, there is always "basis arbitrage" activity in the marketplace. As a result, except for during short-lived periods of stress, the basis is very close to zero.

To summarize, taking a macro-fundamental view of the determinants of CDS premiums, no new information relative to credit spreads is obtained from bonds. The differences between the CDS premiums and credit spreads come from the differences in the institutional features of the CDS and bond markets, liquidity, and the lack of a perfect match between the terms of the two types of instruments.

2.2 Sovereign CDS

Figure 1 displays the historical pattern of the U.S. CDS premiums for the most liquid contracts, which are the five-year swaps. The premiums rapidly increased from 0.2 bps in October 2007 to 20 bps during the Lehman Brothers crisis in September 2008, and continued to escalate until they peaked at 100 bps in March 2009. After the first round of quantitative easing came into effect, the premiums declined and, by October 2009, reached the levels seen during the Lehman Brothers default. Thereafter, the premiums varied between 20 and 65 bps. The premiums started declining around the middle of 2012 and most recently settled at about 20 bps, which is 100 times higher than the pre-crisis level. In Figure 1, we also highlight some of the events

associated with the variations in the cost of protection.

The replication that has been historically applied to corporate CDS is not applicable in this case. Corporate CDS can be valued via replication because the cashflows on a risk-free combination of a bond and its CDS are the same as those on a U.S. Treasury bond of matching maturity if the Treasury bond is risk-free. If a Treasury bond is risky then a risk-free combination of a Treasury bond and its CDS cannot be replicated.

This lack of replication implies that an equilibrium model is needed to determine the CDS premium. An equilibrium setup, which we discuss in the next section, will naturally reveal the potential economic causes of a sovereign credit event. Our primary interest lies in how avenues such as monetary and fiscal policies can trigger a credit event and how the risks of these contingencies are priced.

Failure to pay is another potential trigger of payments on the CDS contracts, which received considerable attention during the congressional debt ceiling debacles of 2011, 2013, and 2015. Many observers believe that one reason for the high U.S. CDS premiums is the chance of default due to the debt ceiling. Indeed, Figure 1 shows that the premiums increased from 40 bps to 60 bps during the first debt ceiling debacle of 2011. However, they declined from 45 bps to 25 bps during the second debt ceiling crisis in 2013, and moved briefly between 15 bps and 25 bps during the 2015 debacle. We find the debt ceiling avenue to be the least interesting economically because it is a hardwired outcome of a political decision-making process (although the state of the economy may influence the specific stances of politicians). Furthermore, recovery is likely to be close to 100% in the case of such a technical credit event, so it is unlikely to have a material impact on the magnitude of the premiums.

There could be non-credit-related risks that we do not account for in our model, but which are potentially responsible for the U.S. CDS premium. First, U.S. CDS are denominated in euros (EUR). The rationale for such a feature is to separate the sovereign risk that the contract insures from the payments made on this contract. Because U.S. Treasuries are denominated in U.S. dollars (USD), the currency of all deliverable bonds is mismatched with the currency of the contracts, which makes it difficult to replicate the U.S. CDS using traded securities. Because the date of a credit event is uncertain, we cannot use a currency forward or swap contracts to perfectly offset the EUR payments with the payments in USD. Although less liquid, trading in USD-denominated contracts started in August 2010 to mitigate this issue. Figure 1 shows the difference between the EUR and USD contracts, and offers a sense of how large the foreign exchange premium can be. It averages 8 bps for the five-year contracts with a standard deviation of 4 bps.

Second, the USD contracts may command a liquidity premium because they are less actively traded. We review a number of measures used to gauge the liquidity of the U.S. CDS market. According to [Augustin \(2014\)](#), with a gross notional sum of \$3 trillion, sovereign CDS constitute about 11% of the overall credit derivatives market. Dealers have the largest market share of 70%. In particular, the average gross (net) notional amount of outstanding U.S. CDS is \$17 (\$3.2) billion. To gain further insight into the trading activity of the U.S. CDS, we report our crude measure of liquidity in Figure 2. Because the contracts on Italian government debt are the most actively traded sovereign CDS, we report the ratio of the weekly net notional amount of U.S. CDS to that of Italian CDS.¹ The average ratio is 18% and the ratio ranges between 6.5% at the beginning of the sample in 2008 to 33% in late 2011 at the

¹We are indebted to Patrick Augustin for sharing this data, which he hand-collected from the Depository Trust and Clearing Corporation.

peak of the anxieties regarding the European credit crisis and U.S. fiscal uncertainty. So, although the U.S. contracts are clearly not the most liquid, they are nonetheless heavily traded.

Third, the Basel III capital charge rule may affect the magnitude of the CDS premium even when there is absolutely no credit risk. Dealers are allowed to buy protection against sovereign default to reduce the capital charge associated with their counterparty risk exposure. As pointed out by [Klingler and Lando \(2015\)](#), a sovereign protection seller requires a positive CDS premium even when the sovereign is riskless because of the capital constraints. Anecdotally, some dealers began to implement the rule voluntarily in 2013. [Klingler and Lando \(2015\)](#) empirically attribute a fraction of CDS premiums to this effect in their sample from 2010 to 2014.

Fourth, there is legal risk associated with the determination of a credit event by a committee comprised of 15 voting members: 10 from the sell side and five from the buy side. At present, there is poor understanding of the incentives of the committee participants and how they may affect the decision on whether a credit event has occurred. Last, but not least, there is a risk of uncertain recovery that is determined by a bond auction with a cheapest-to-deliver option.

Fifth, there is a counterparty credit risk that could be reflected in the CDS premiums. We believe that because of the full collateralization, this risk is minimal. Nevertheless, this counterparty risk can manifest itself in two related ways. First, if one of the parties defaults, the contract is extinguished. Thus, if the remaining party wants to re-establish a similar position with someone else, it faces a rollover risk in that it might not be able to do so at the last recorded marked-to-market value. Second, the remaining party has the right to seize the collateral, but might not be

able to dispose of the collateral immediately at marked-to-market values. [Arora, Gandhi, and Longstaff \(2012\)](#) study the impact of counterparty risk on corporate CDS valuation concluding that “... the magnitude of the effect ... is vanishingly small.”

3 The Model

In our model, we use a standard framework to link the nominal debt, taxes, inflation, and aggregate growth to the fiscal and monetary policies through the government’s budget constraints. The government can maintain the budget balance either by issuing new debt or raising inflation or taxes. Fiscal default arises when the government can no longer service its debt, rendering it insolvent. As a result, investors may want to buy protection against default events through sovereign CDS contracts.

As we pointed out earlier, we cannot use the standard replication argument to value CDS when treasuries are themselves subject to credit risk. We therefore complement our setup with a representative investor with [Epstein and Zin \(1989\)](#) preferences who uses her marginal rate of substitution to value assets. This allows us to value any financial security.

In this section, we describe the details of our model. We start with the pricing kernel, which we derive from the representative investor’s preferences and her aggregate consumption process. Next, we describe the dynamics of the aggregate economy and government. Then we specify the interaction of the government’s fiscal and monetary policy stances with the real economy. We conclude with the valuation of defaultable securities such as CDS.

Notation. We use capital letters to denote the levels of the variables. Lowercase letters are used for their logs. The changes in the variables are denoted by Δ .

3.1 Valuation of Financial Assets

We assume a representative agent with recursive preferences:

$$\begin{aligned} U_t &= [(1 - \beta)C_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho}, \\ \mu_t(U_{t+1}) &= E_t(U_{t+1}^\alpha)^{1/\alpha}, \end{aligned}$$

where $\rho < 1$ captures time preferences (intertemporal elasticity of substitution is $(1 - \rho)^{-1}$), and $\alpha < 1$ captures the risk aversion (relative risk aversion is $1 - \alpha$). Aggregate consumption is denoted by C_t .

With this utility function, the real pricing kernel is:

$$M_{t+1} = \beta(C_{t+1}/C_t)^{\rho-1}(U_{t+1}/\mu_t(U_{t+1}))^{\alpha-\rho}.$$

In our model, we assume the economy is cashless and we use money as a unit of account only. Correspondingly, P_t denotes the price level. The agent uses the nominal pricing kernel $M_{t+1}^s = M_{t+1}\Pi_{t+1}^{-1}$, where $\Pi_t = P_t/P_{t-1}$ is the inflation rate, to value nominal assets. We provide the determinants of endogenous inflation below.

Consumption is assumed to have the following dynamics:

$$\begin{aligned} \Delta c_{t+1} &= \nu + x_t + \sigma_c \varepsilon_{t+1} \\ x_{t+1} &= \varphi_x x_t + \sigma_x \varepsilon_{t+1}, \end{aligned}$$

where the shock ε_{t+1} is $\mathcal{N}(0, 1)$. This assumption is similar to [Bansal and Yaron \(2004\)](#), Model I, by allowing for a time-varying conditional mean in consumption growth. The shock to consumption growth and its expectation are perfectly correlated for simplicity. Parameter ν captures the deterministic trend growth rate.

Although it is reasonable to expect that a U.S. default would affect the representative agent’s consumption, we do not allow for this in our model for two reasons. First, we intentionally restrict ourselves to a well-accepted real pricing kernel that has been implemented in a number of asset-pricing applications. Our contribution is in assessing whether the existing modeling elements that are combined so that they can speak to the fiscal environment in the U.S. can give rise to the observed CDS premiums. Second, given the wide latitude for modeling the impact of U.S. default consumption, a general equilibrium approach needs to be implemented to discipline the model. However, such a model is outside of the scope of this paper.

3.2 The Government and the Economy

We assume that output Y_t evolves as follows:

$$\Delta y_{t+1} = \nu + \varphi_y(\tau_t - \tau) + \sigma_y \varepsilon_{t+1}, \tag{1}$$

where $\tau_t = \log \mathcal{T}_t$ is the (log) tax rate at time t and τ is its unconditional mean. The trend growth rate of output growth is set to that of consumption growth, ν , to ensure a balanced growth path. We assume the existence of a single tax rate and remain agnostic about its precise nature. The tax rate is time-varying and its dynamics arise endogenously through the fiscal authority’s response to debt, as

specified below.² Identical shocks to output and consumption serve as a modelling shortcut to the resource constraint that arises in general equilibrium models.

Importantly, we assume that deviations of the prevailing tax rate from the mean affect future growth prospects, through the parameter φ_y . Consistent with the literature (Croce, Kung, Nguyen, and Schmid, 2012; Jaimovich and Rebelo, 2012), φ_y is negative and small in our calibration, so raising taxes depresses future growth prospects. Although we assume this link directly, our specification is in the spirit of the literature on endogenous growth and taxation in which an elevated tax burden endogenously decelerates growth through its effect on innovation (Rebelo, 1991; Croce, Nguyen, and Schmid, 2013).

Let G_t be the government expenditure as a fraction of output. Its log dynamics are given as follows:

$$g_{t+1} = (1 - \varphi_g)g + \varphi_g g_t - \sigma_g \varepsilon_{t+1}.$$

The minus sign in front of the volatility coefficient σ_g highlights the perfect negative correlation between the shocks to output and expenditure, so that a bad shock to the economy corresponds to an increase in expenditure.

To finance its expenditure, the government raises taxes and issues nominal debt. For simplicity, we assume that the government directly taxes output, so that the tax revenue at time t is given by $\mathcal{T}_t Y_t$. We view this specification as a tractable way to capture the link between taxation and the aggregate economy. We assume that the government issues nominal debt with a face value of N_t . The real face value of debt

²One concern is that the economy attains infinite output as the tax rate approaches zero. In practice, such a scenario is not feasible due to the endogenous nature of taxes in our model.

as a fraction of output is:

$$B_t = (N_t/P_t)/Y_t.$$

The government finances its expenditure with two types of bonds: short-term with a price of Q_t^s and long-term with a price of Q_t^ℓ per \$1 of face value. Short-term bonds mature in one period. We consider the short-term bonds to be a type of monetary policy instrument. We model long-term debt to allow for more realistic modeling of default and to be able to account for the quantitative easing period within the context of our setup. For tractability, we assume that short- and long-term bonds are issued in constant proportion: the nominal amounts are $N_t^s = \omega N_t$ and $N_t^\ell = (1 - \omega)N_t$, respectively. Variation in ω can represent shifts in the overall maturity structure of government debt held by the public, such as those induced by the Federal Reserve's quantitative easing program. We explore these variations later in the paper.

To retain a stationary environment with long-term debt, we model the environment via a sinking fund provision in the spirit of [Leland \(1994\)](#). A long-term bond specifies a coupon payment γ every period and requires a fraction λ of the debt to be repaid every period. This amounts to a constant amortization rate of the bond. Although this is perpetual debt, it has an implicit maturity that is determined by the repayment rate λ . If $\lambda = 1$, then Q_t^s and Q_t^ℓ have the same maturity; if $\lambda < 1$, then the implicit bond maturity is longer and proportional to $1/\lambda$.

In the absence of default, the properties of debt and taxes are connected via the

GBC:

$$\begin{aligned} \mathcal{T}_t Y_t + Q_t^\ell (N_t^\ell - (1 - \lambda)N_{t-1}^\ell) / P_t + Q_t^s N_t^s / P_t \\ = (\gamma + \lambda)N_{t-1}^\ell / P_t + N_{t-1}^s / P_t + G_t Y_t. \end{aligned} \quad (2)$$

The GBC requires that government expenditures $G_t Y_t$ and due payments on short- and long-term debt (coupon payments and amortization) $(\gamma + \lambda)N_{t-1}^\ell / P_t + N_{t-1}^s / P_t$ have to be covered either by tax income $\mathcal{T}_t Y_t$ or by issuing new short- or long-term debt $Q_t^\ell (N_t^\ell - (1 - \lambda)N_{t-1}^\ell) / P_t + Q_t^s N_t^s / P_t$. The GBC implies the following tax rate:

$$\mathcal{T}_t = G_t - Q_t B_t + CF_t B_{t-1} \Pi_t^{-1} (Y_t / Y_{t-1})^{-1}, \quad (3)$$

where $Q_t \equiv \omega Q_t^s + (1 - \omega)Q_t^\ell$ is the market value of one unit of the debt-to-GDP ratio, and $CF_t \equiv \omega + (1 - \omega)(\gamma + \lambda + (1 - \lambda)Q_t^\ell)$ is the promised cash flow per one unit of debt.

We capture the monetary and fiscal policy stances by means of policy rules. In the case of the monetary policy, this is achieved by a standard Taylor rule linking the nominal short-term interest rate to macroeconomic variables. In line with the literature, we assume that the central bank responds to inflation and output growth, which we view as corresponding to the output gap in the new-Keynesian literature.

In the case of fiscal policy, we assume that the government sets the amount of new debt issued in response to the amount of debt outstanding and expected economic conditions x_t . Then, the prevailing tax rate has to be such as to establish a budget balance in the GBC, as shown in equation (3).

Our specification is related to policy rules examined in the recent literature on

monetary-fiscal interactions (Bianchi and Ilut, 2014; Leeper, 1991, 2013; Schmitt-Grohe and Uribe, 2007). In particular, Schmitt-Grohe and Uribe (2007) has shown that in a rich new-Keynesian dynamic stochastic general equilibrium model, policy rules of this sort lead to welfare levels that are quantitatively indistinguishable from those stemming from optimal Ramsey policies, in which the fiscal and monetary policies are designed to maximize social welfare. Relatedly, Cuadra, Sanchez, and Sapriza (2010) show how debt and tax dynamics that are consistent with our specification arise endogenously in Ramsey optimal fiscal policies when the government has the option to default.

Summarizing, the government controls the real debt and nominal interest rate through the fiscal and monetary policies, respectively, as follows:

$$\begin{aligned}
 b_t &= \rho_0 + \rho_b b_{t-1} + \rho_x x_t, && \text{(Fiscal policy)} \\
 -q_t^s &= \delta_0 + \delta_\pi \pi_t + \delta_y \Delta y_t, && \text{(Monetary policy)}
 \end{aligned}$$

where $\pi_t = \log \Pi_t$ is the (log) inflation rate. Intuitively, the parameter ρ_b determines how fast the government intends to pay back the outstanding debt. Similarly, we allow for the possibility that the government increases public debt in bad times by responding to x_t . The parameter $\rho_x < 0$ determines the intensity of this interaction.

Given the real pricing kernel, the Taylor rule and the Euler equation imply the dynamics of inflation as in Gallmeyer, Hollifield, Palomino, and Zin (2007). This reflects the requirement that the nominal short rate, that is computed using the nominal pricing kernel (and the default-contingent payoff), and the one dictated by the Taylor rule must coincide in equilibrium. In this line of work, which revolves around endowment economies with fully flexible pricing mechanisms, monetary pol-

icy has no capacity to affect real variables. In our setting, the GBC is the channel through which monetary policy influences real quantities because it affects the real value of outstanding debt, which in turn impacts the tax rate and output growth.

3.3 Fiscal Default

We consider government default in the model to be a form of fiscal default, or in other words as scenarios in which the budget balance can no longer be restored by further raising taxes, as opposed to mere technical defaults resulting from the political decision-making process. Our model captures the negative effect of taxes on the tax base by means of the output growth equation (1). This effect limits the future stream of surplus the government can generate in any state, and thus the maximal amount of debt it can repay.

Limits to raising taxes arise frequently in macroeconomic models with distortionary taxes in the context of Laffer curves. Laffer curves relate the government’s tax revenue to the prevailing tax rate. Although they typically start out increasing for low tax rates, they can reach the “slippery slope” (Trabandt and Uhlig, 2011) where raising the tax rates actually lowers the tax revenue. In this scenario, tax policy becomes an ineffective budget-balancing tool because the distortionary taxation tends to negatively affect the tax base, such as in the case of labor taxes, where excessive taxation reduces work incentives.

To capture this Laffer curve intuition, we introduce two notions of expected surplus.

One is the present value of tax receipts minus government expenditure:

$$S_t = E_t \sum_{j=1}^{\infty} M_{t,t+j} (\mathcal{T}_{t+j} - G_{t+j}) Y_{t+j} / Y_t. \quad (4)$$

Note that S_t only coincides with the market value of debt, $Q_t B_t$, when there is no default. The second notion is the expected sustainable surplus, which corresponds to the maximal tax rate, \mathcal{T}_t^* , that is feasible without lowering tax revenues:

$$S_t^* = E_t \sum_{j=1}^{\infty} M_{t,t+j} (\mathcal{T}_{t+j} \wedge \mathcal{T}_t^* - G_{t+j}) Y_{t+j}^* / Y_t^*,$$

and \mathcal{T}_t^* solves $S_t = S_t^*$. The notation Y_t^* highlights the different dynamics of output if the tax rate changes from the one prescribed by the GBC. If $\mathcal{T}_t > \mathcal{T}_t^*$, then the shrinking tax base will decrease the surplus.

These equations capture the idea that if \mathcal{T}_t becomes greater than \mathcal{T}_t^* , then the current government policies are not sustainable. So, the government should either adjust one of its policies, or default. We assume that the government is committed to its expenditure and the monetary and debt management rules. Expenditures reflect, to a large extent, various entitlement programs that are hard to renegotiate. We intentionally do not allow for changes in the policy rules. By doing so, we effectively assume that the Fed will never be insolvent separately from the Treasury, that is, the Fed has the *fiscal support* of the Treasury (Reis, 2015). These assumptions allow us to highlight the default channel of CDS premiums. In practice, many changes may take place in extreme fiscal situations. Studying all of the possibilities is beyond the scope of this paper.

Indeed, if \mathcal{T}_t becomes greater than \mathcal{T}_t^* , the expected surplus required to service

the debt exceeds the surplus that the government can sustain by committing to its policy rules. In this case, the government will no longer be able to honor its long-term financial obligations. At this stage, rational investors will not be willing to roll over the short-term debt. Unable to access the bond market, the government has to default.

The fiscal theory of the price level also features a prominent role for the GBC in a similar situation of fiscal stress, in which the GBC is required to hold. As a result, the price level P_t is determined via the equality of the market value of debt, $Q_t B_t$, and the expected surplus (4). [Cochrane \(2011\)](#) points out that in this case reaching the top of the Laffer curve leads to fiscal inflation instead of default. As [Leeper \(1991\)](#) and [Woodford \(2003\)](#) show, such a mechanism leads to a determinate equilibrium only when the fiscal policy is active (locally non-Ricardian) and the monetary policy is passive.

In our model, the monetary policy rule satisfies the Taylor principle implying a unique bounded path for inflation, P_t/P_{t-1} . The fiscal policy operates on the stock of government debt and with $\rho_b < 1$ ensures a unique bounded path for this stock. Thus, both policies are active, so they are uncoordinated ([Cochrane, 2011](#)). This is feasible because we allow for default via violation of the GBC.

The scenario outlined in our setup is unusual vis-à-vis the historical experience, which is primarily related to emerging economies. It is common to model sovereign default in these economies as a strategic choice. In our setting, the government has complete control of its monetary policy, all government debt issued in its own currency, and, yet, default is not its choice. Unfortunately, we do not have evidence for developed economies to justify this framework based on realizations. That is why

we are using default-sensitive securities to infer market perceptions of the likelihood of such outcomes.

3.4 Defaultable Securities

We denote the default time by:

$$t^D = \min\{t : \tau_t^* \leq \tau_t\},$$

and the probability of default by P_t^D . So a default will occur at time $t+1$ if $t^D = t+1$. Given the definition of the one-period ahead default probability, we can value the short-term bond as:

$$Q_t^s = E_t \left(M_{t,t+1}^{\$} \left[(1 - \mathbf{1}_{\{t^D=t+1\}}) + (1 - L)\mathbf{1}_{\{t^D=t+1\}} \right] \right), \quad (5)$$

where L is the loss given default. We can also value the long-term bond by relying on the one-period ahead default probabilities via the following recursive representation:

$$Q_t^\ell = E_t \left(M_{t,t+1}^{\$} \left[(\gamma + \lambda + (1 - \lambda)Q_{t+1}^\ell)(1 - \mathbf{1}_{\{t^D=t+1\}}) + (1 - L)\mathbf{1}_{\{t^D=t+1\}} \right] \right).$$

A CDS contract has two legs: the premium leg pays the CDS premium CDS_t^T every quarter until a default occurs, and pays nothing after default. The protection leg pays a fraction of the face value of the debt that is lost in default and nothing if there is no default before maturity. Accordingly, the value of the fixed payment to be made at time $t + j$ is $CDS_t^T \times E_t(M_{t,t+j}^{\$} \mathbf{1}_{\{t+j < t^D\}})$. As a result, the value of the

premium leg is equal to:

$$\text{Premium}_t^T = CDS_t^T \cdot \sum_{j=1}^T E_t M_{t,t+j}^{\$} \mathbf{1}_{\{t+j < t^D\}}.$$

The protection leg can be represented as a portfolio of securities, each maturing on one of the days of the premium payment, $t + j$, and paying L if a default occurs between $t + j - 1$ and $t + j$, and nothing otherwise. Thus,

$$\text{Protection}_t^T = L \cdot \sum_{j=1}^T E_t (M_{t,t+j}^{\$} \mathbf{1}_{\{t+j-1 < t^D \leq t+j\}}).$$

The CDS premium CDS_t^T is determined by equalizing the values of the two legs.

Importantly, CDS premiums depend on the joint behavior of the nominal pricing kernel and default probabilities. Although we specify the process for the real pricing kernel exogenously, the default probabilities reflect the endogenous responses of our economy to shocks. To the extent that the endogenous dynamics of our economy are predictive of high government indebtedness in times of low consumption growth prospects, the representative agent in our model will require compensation for the potential default losses during such periods. In other words, the prices of the default-sensitive securities reflect a risk premium beyond the expected losses.

3.5 Discussion

In our simple model of the U.S. economy, we allow for scenarios that endogenously trigger the government to default on its debt. Before we describe the model's solution, we briefly review its ingredients.

There are four building blocks. In the first block, we describe the dynamics of the aggregate economy as given by (1). In the second block, we outline the government-related objects, such as the fiscal and monetary rules, and the GBC. In the third block, we describe the default condition that is based on the Laffer-curve argument. Finally, in the fourth block we derive a risk-sensitive pricing kernel from the recursive preferences given a process for consumption growth. Although blocks one and four reflect a standard structure in the literature on long-run risks following [Bansal and Yaron \(2004\)](#), we add a specification of the government and the central bank policy instruments and a default event in blocks two and three. Although we do not complete the model in general equilibrium, we link all of these blocks through the GBC.

Inflation arises endogenously because the nominal interest rate implied by the Taylor rule has to coincide with that implied by the nominal pricing kernel. Inflation thus has real effects in our model because it affects the real value of debt and thus the prevailing tax rate, which in turn affect the expected growth. Growing debt-financed government deficits can lead to periods of elevated tax rates, which may trigger default.

Default probabilities are reflected in the pricing of defaultable bonds. Treasury bonds and thus the central bank's policy instrument are themselves subject to credit risk. Even the value of a hypothetical nominal bond that has no cash flow risk depends on the default probability because the combination of fiscal and monetary policies and the GBC imply that inflation depends on the risk of government debt.

4 Quantitative Analysis

In this section, we evaluate the extent to which the possibility of a U.S. fiscal default can quantitatively account for the CDS premiums observed since the onset of the recent financial crisis. We benchmark the baseline calibration of our model to salient features of the recent U.S. monetary and fiscal environment, consistent with the timeframe when U.S. sovereign CDS are traded in the market place. In this setting, we check whether the calibrated model implies CDS premiums consistent with the ones observed in the data. Furthermore, our risk-sensitive specification allows for a decomposition of CDS premiums into a default probability and a default risk premium. We can also use our calibrated model as a laboratory for a set of counterfactual experiments that highlight the different channels that affect the valuation of sovereign default risk. We start by describing our simulation and calibration approaches, and then illustrate the main mechanisms driving the quantitative results and counterfactuals.

4.1 Numerical Approach

Our quantitative results are based on model simulations. The possibility of default induces strong nonlinearities in both payoffs and the discount factor. Therefore, we use a global, nonlinear solution method. Endogenous variables are approximated as functions of the state variables for all values using Chebyshev polynomials on grids. We use projection methods to solve for these approximations. The procedure is outlined in [Appendix A](#).

In particular, in contrast to the more familiar perturbation techniques, in which

endogenous variables are approximated by expansions around a particular point in the state space, our approximation does not deteriorate when we move away from the steady state, even to extreme regions of the state space, precisely because Chebyshev polynomials are globally defined.

The usual challenge in nonlinear models such as ours is to ensure that a unique and accurately computed stationary solution exists. Our numerical approach is capable of addressing this concern. Indeed, by virtue of our global solution technique, it is straightforward to verify that all of the endogenous variables are stationary across the entire state space for a given set of parameters. The global approximation ensures the accuracy of the approximation to the desired degree of precision regardless of the distance from the steady state.

The uniqueness of the solution is a more challenging issue, for two reasons. First, it is common to use the Blanchard-Kahn conditions applied to log-linearized models to analytically verify a solution's uniqueness. However, this technique is not applicable in our case precisely because it quickly loses precision due to the existing non-linearities. Second, and more broadly, it is typical for models such as ours in which default is tied to endogenous valuations to exhibit multiplicities, see, e.g., [Aguiar and Amador \(2018\)](#) and [Crouzet \(2017\)](#). In this context, our global solution technique allows for the exclusion of non-stationary equilibria. How to establish constraints on the selection of equilibria in non-linear models with default remains an important open question.

4.2 Calibration

We report our baseline parameter choices in Table 1. We calibrate the model at a quarterly frequency, consistent with the availability of macroeconomic data. We need to calibrate parameters from four groups. First, we follow the literature on long-run risks to select our preference parameters. Second, we pick parameters governing the exogenous stochastic processes in our model, such as output growth, consumption growth, and, critically, government expenditure. We do so by matching time series moments of their empirical counterparts in the postwar sample, 1947-2016. Third, we choose parameters that control the maturity and payment structure of government debt. Finally, we specify the fiscal and monetary policy rules to match the recent U.S. policy environment in a high debt setting. We remove the deterministic trend by setting $\nu = 0$.

Our choice of preference parameters follows [Bansal and Yaron \(2004\)](#). As is well-known, the combination of relatively high risk aversion and an intertemporal elasticity of substitution above one allows the rationalization of sizeable risk premiums in many markets. In a similar vein, the calibration of the consumption growth process reflects the long-run risks, and the parameter choices follow [Bansal and Yaron \(2004\)](#), converted from monthly to quarterly frequency. The only difference is that in our model the innovations to consumption growth and its conditional expectations are perfectly correlated. This leads to a modest increase in maximum Sharpe ratio, computed as conditional volatility of log real pricing kernel, from 0.16 to 0.22.

To calibrate G_t , we fit an autoregressive process to the GDP-government expenditure ratio, which helps us to determine its mean, autocorrelation, and volatility. For the output dynamics, a critical parameter is φ_y , which is the elasticity of the output

growth with the respect to taxes. Intuitively, we would expect an increase in taxation to be bad news for trend growth. By setting $\varphi_y = -0.024$, based on the empirical estimate obtained in [Croce, Kung, Nguyen, and Schmid \(2012\)](#), our parameter choice is consistent with this notion. We choose σ_y to match the relative volatility of the consumption and output growth observed in the data.

Our setting is unusual because there is only a short period of data on our primary object of interest, the U.S. CDS, during a particular economic period, namely 2007-2016. In particular, the debt-to-GDP ratio, which is instrumental for determining the likelihood of fiscal stress, exhibits drastically different properties during the post-war and post-financial crisis periods. Indeed, a structural break test identifies that the mean of the ratio changes twice, in the mid-1980s from 0.35 to 0.57 and immediately before the crisis from 0.57 to 0.9.

To examine the specific sample and the broader economic mechanisms, we follow the macroeconomic literature and conduct transitional dynamics exercises ([King and Rebelo, 1993](#)). The model is calibrated to match the post-war sample debt-to-GDP mean of 0.57. Then, for our headline results, we start the economy at the debt-to-GDP value selected to match the average debt levels during the recent period (2007-2016). Subsequently, we conduct a counterfactual analysis of hypothetical CDS premiums in the full post-war sample starting the economy at a different debt-to-GDP value that is selected to match the average debt levels during the full sample. These conditional economies converge to the calibrated steady-state economy for a large number of observations.

The weighted average maturity of U.S. Treasury bonds is 59 months. However, the figure has risen consistently over the past few years, reaching about 69 months at

the end of 2015 (Treasury, 2010). In addition, debts that mature in less than one year represent about 20%-30% of all outstanding debt. These numbers allow us to select the (ω, λ) combination. We pick ω to be 0.2 to match the latter figure. To match the long-term average maturity, we select $\lambda = 0.04$. Finally, there is little guidance about the recovery rate in the event of a U.S. government default. Perhaps erring on the conservative side, we assume a recovery rate of 70% ($L = 0.3$) in our benchmark calibration. This value is at the higher end of the recent estimates across the Eurozone countries by Augustin, Chernov, and Song (2018) and quite a bit higher than in the U.S. corporate bond market, where recovery rates of around 50% are a good starting point, as reported, for example, in Chen (2010).

Our calibration of the parameters in the policy rules follows a standard approach. We choose the parameters of the Taylor rule following the parameterization in Gallmeyer, Hollifield, Palomino, and Zin (2007), which implies an average inflation rate in line with the data. To determine the parameters in the fiscal rule, we run a regression of the debt-to-GDP ratio on its lagged value and a proxy for expected consumption growth. We compute an estimate of x_t from the data on consumption growth using the Kalman filter and the assumed model parameters.

4.3 Quantitative Results for the Baseline Sample

We now present our quantitative results based on the model simulations. In this section our objective is to evaluate the extent to which a model based on the notion of fiscal default can account for the CDS premiums observed between 2007 and 2016. We start by discussing the macroeconomic implications of the model. Taking these as a benchmark, we proceed to examine the quantitative implications for CDS

premiums.

4.3.1 Matching Quantities

In Table 2, we summarize the main implications for the macroeconomic quantities for the baseline sample. Here, we start our simulations in a region of the state space that approximates the recent fiscal environment. The average market value of debt to GDP ratio in the model is about 0.9, which is within one standard error of the figure in the data. Identifying and determining a single relevant aggregate tax rate is complicated by the tax code. We use the estimates from [McGrattan and Prescott \(2005\)](#) as our sample statistics. Our model matches these numbers quite closely. The average inflation is also matched.

Although matching the basic macroeconomic moments is important to discipline our analysis, our main interest is the potential for fiscal default. The results in Table 2 give a sense of the possibility of such an event in our model. The results suggest that the unconditional mean of the debt limit is in the range of a 130% to 170% percent debt-to-GDP ratio. These numbers are well within the range of the CBO long-term debt projections ([CBO, 2016](#)). The corresponding tax limit is 70% to 97%. These are large numbers compared to the average current tax rate. Yet the low bound is not far from that of [Trabandt and Uhlig \(2011\)](#). Moreover, we would expect the debt and tax limits to fall during economic downturns. We confirm this intuition below.

The estimated distribution of the tax limit determines the fiscal default probabilities in the model. Our benchmark calibration yields a one-year ahead default probability of 0 to 0.3%. As an external validation of this range, Moody's estimates this prob-

ability at 0.05% (Tempelman, 2011). Below we explore the extent to which such a default probability can account for the observed CDS premiums.

4.3.2 Inspecting the mechanism

To dissect the main economic mechanism underlying our quantitative results, we inspect the response of our economy to a negative one-standard deviation shock to the long-run trend, x_t . In our model, changes in the variables are perfectly correlated, as is the case in a general equilibrium environment. Thus, the behavior of the variables is driven by the properties of the long-run trend. Figure 3 illustrates the comovement of all our variables. The same patterns are also reflected in the unconditional correlations reported in Table 3.

A negative shock to the long-run consumption trend triggers an increase in government expenditure, which is consistent with the counter-cyclicality of government expenditure. Naturally, this increased expenditure gives rise to financing needs. Our fiscal policy rule then requires that the expenditure is partially financed by the government issuing debt. Due to the fiscal rule, the government debt is realistically countercyclical. However, the GBC requires a balanced budget, so the increased expenditure also leads to a rise in the tax rates. Our specification of the fiscal block of the model is thus consistent with countercyclical fiscal policies.

We now examine how the fiscal and monetary policies interact in our model. First, given our specification of the process governing output growth, a higher tax rate reduces the expected output growth. As a result, an accommodative central bank will try to stimulate the economy by lowering the nominal short rate, thereby creating

inflation (inflation is countercyclical). This would not be the case if the bank follows a simple Taylor rule responding to inflation only.

The dynamic behavior of inflation deserves particular attention. Although the unconditional covariance of consumption growth with inflation in the data is consistent with the implications of our model, recent research has challenged the notion of countercyclical inflation on the basis of sub-samples. For example, [Campbell, Pflueger, and Viceira \(2018\)](#) and [Song \(2017\)](#), report that the relevant correlation was positive in the post-financial crisis episode. We find a correlation of 0.1 between consumption growth and inflation from 2007Q1 to 2015Q4, while we obtain a correlation of -0.09 in the postwar sample from 1947Q1 to 2015Q4. Moreover, inflation expectations have been puzzlingly low recently in spite of expansionary monetary policies in the US.

Current macroeconomic modelling lacks a framework that accommodates this ‘missing inflation puzzle’, as discussed, for example, in [Arias, Erceg, and Trabandt \(2016\)](#). Because our setup is intentionally reliant on standard modeling tools, it is subject to the same critique. However, extending the model to multiple regimes is beyond the scope of this paper. We conjecture that accounting for falling and persistently low inflation in severe downturns, while challenging in our modeling framework, would likely strengthen our results, as it would raise the government’s real debt burden. That is reminiscent of the classic Fisherian ‘debt deflation’ effect (see, for example, [Gomes, Jermann, and Schmid, 2016](#) for a recent interpretation).

Because the tax base shrinks when taxes go up, and government expenditures increase persistently, the fiscal limit (maximal sustainable tax rate) declines and the default probability increases. Note that this increase in the default probability coincides with

an upward jump in the stochastic discount factor, or marginal utility. In other words, in our model, periods with high default probabilities and thus high potential losses endogenously coincide with times of high marginal utility. To bear the risk of such losses, the agents in our model thus require a credit risk premium to hold defaultable securities. This credit risk premium allows our model to generate non-trivial CDS spreads.

4.3.3 Term Structures of Risk-free and Defaultable Securities

As discussed above, the standard replication approach for corporate CDS contracts does not apply in the context of U.S. sovereign CDS premiums due to the lack of a risk-free benchmark. Although U.S. Treasury bonds are often conveniently interpreted as such a benchmark, the notion of observed non-zero CDS premiums on U.S. government debt invalidates this view.³ When U.S. government debt is also subject to credit risk, approaches other than replication are called for when determining the CDS premiums. Our equilibrium model offers such an approach.

The pricing kernel in our model implies an equilibrium term-structure of real risk-free yields. However, the term structure of U.S. Treasury yields cannot serve as an empirical counterpart to these yields. There are two sources of discrepancies. First, the term structure of Treasury yields refers to nominal bonds. Second, and more importantly, these bonds are not insulated from the credit risk, as highlighted above. Nonetheless, we can infer a theoretical counterpart to U.S. Treasury yields

³ The prevailing post-crisis reference interest rate curve based on overnight index swaps is also not risk-free because it is based on the Federal Fund rate, and, as discussed in section 3.3, the Fed is never insolvent separately from the Treasury.

from our model by using the nominal pricing kernel and accounting for a possibility of default similar to expression (5).

In Table 4, we summarize three yield curves inferred from our calibrated model. We show the term structure of risk-free yields that correspond to expectations of the equilibrium *real* pricing kernel at various horizons. We report what we call the term structure of pseudo risk-free nominal yields. The curve corresponds to expectations of the equilibrium *nominal* pricing kernel. We label them pseudo risk-free, because the endogenous inflation in our model reflects the risk of a government default, while the real discount factor does not. We also report the yield curve of nominal, defaultable bonds that correspond to expectations of the nominal pricing kernel accounting for the government default probabilities at various horizons, that is, the term structure of *default probabilities*.

The term structure of real risk-free yields is mildly downward sloping, which is consistent with the long-run risk paradigm. In the context of our model, this is an implication of a high intertemporal elasticity of substitution. Empirically, a clear consensus about the average slope of the real term structure has not yet emerged. Various researchers have interpreted the data on inflation-protected bonds in the U.S. as pointing to an upward sloping real yield curve, while others point to the short data sample and conflicting evidence from a longer data sample on inflation-indexed bonds in the U.K. Neither line of argument provides guidance for our purposes, because even an upward sloping term structure of real yields does not allow the effects of inflation and default risk to be disentangled, which is at the core of our setup.

Given the real risk-free term structure, our model generates nominal pseudo risk-free yield curves that are on average upward sloping. Thus, our model predicts a real-

istically upward sloping term structure of inflation expectations and, importantly, inflation risk premiums. As described earlier, inflation is endogenously countercyclical in our model. Indeed, adhering to the Taylor rule requires the central bank to raise inflation in response to elevated government debt to restore the budget balance by eroding the real debt burden. This move is necessary because high debt leads the government to raise taxes, which typically lowers the long-term growth prospects and output growth, which the central bank reacts to. Inflation thus erodes the payoff from holding debt in high marginal utility periods, so that bond holders will require an inflation risk premium to hold government bonds.

We find that the term structure of the nominal defaultable yields – the model counterpart of the U.S. Treasury yield curves – is also upward sloping. This curve reflects the inflation expectations and an inflation risk premium adjustment, and it also accounts for the term structure of the default expectations and a *default risk premium*. The default risk premium accounts for the fact the model is naturally predictive of potential government defaults that may occur during high debt periods, which we show to endogenously coincide with the periods of high marginal utility in the model. Notably, the defaultable term structure is steeper than the nominal pseudo risk-free curve, implying that default risk cannot be avoided by inflating away debt. These default premiums thus reflect the market expectations about the central bank’s capacity to restore the budget balance by means of inflation, which is consistent with the empirical evidence in [Hilscher, Raviv, and Reis \(2014\)](#) on the limited ability of inflation to balance the government budget. We explore this further in the counterfactual analysis.

4.3.4 Fiscal Defaults and CDS Premiums

We now examine the pricing of CDS contracts and the link between CDS premiums and the probability of a U.S. fiscal default. Table 5 provides the results. We report the average CDS premiums from the data and the model in basis points. Columns (1) to (3) report various versions of the data depending on the contract denomination (EUR in (1) and (2), and USD in (3)), and sample (2007-2016 in (1), and 2010-2016 in (2) and (3)). The main differences in the averages are driven by the currency of the contract denomination rather than the sample.

The model-based averages are reported in column (4) of Table 5. Overall, we can see that our model delivers an upward sloping term structure of CDS premiums with magnitudes that are consistent with the data. The magnitudes are particularly close to the USD-denominated numbers, which is natural because we ignore the currency risk in our model. Although there are some quantitative discrepancies, as highlighted in section 2.2, we do not account for the risks associated with the various institutional features of the contracts in our model. On balance, the results suggest that accounting for the default risk goes a long way toward explaining the magnitudes of the CDS premiums.

Traditionally, credit models better fit premiums at the longer end of the curve than at the short end. This is a standard implication of structural models of the defaultable term structures, especially those that are consumption based. We find magnitudes that are consistent with the data because the model's default boundary fluctuates over time and time is discrete, so investors cannot perfectly anticipate default in the next period.

The CDS premiums we find are substantial despite the modest default probabilities

in the model. As in all models of defaultable securities, default spreads can be decomposed into two components: expected losses and default risk premiums. In our model, the losses given default are known, so the default risk premium reflects the compensation protection that sellers require to bear the risk of experiencing a default during a period of high marginal utility. The calibrated loss is relatively small because the burden of fitting the CDS premiums rests on the ability of our model to generate high default risk premiums.

Indeed, the results of column (5) of Table 5 confirm that the risk premiums are substantial. The column displays the size of the CDS premiums when investors are risk-neutral. The ratio of the numbers in column (4) to the ones in column (5) reflects the magnitude of the aforementioned default premium. The ratio ranges between 2 and 3 across all maturities. The default premium is large because fiscal default is more likely to happen endogenously during periods of high marginal utility, so that selling default insurance earns a high covariance risk premium, akin to the default risk premia in other debt markets, such as corporate bond markets. Thus, the model is consistent with high CDS premiums, which reflect investors' rational forecasts of the likelihood of U.S. fiscal stress.

4.3.5 Dynamic implications

It is straightforward to extract the historical shocks that have driven the U.S. economy in the postwar period and study the dynamic implications of our model. Indeed, through the lens of our single factor economy, data on consumption growth is sufficient to determine the underlying shocks. We feed this data into our model and compare the statistical properties of the resulting time series with those of the his-

torical series.

Direct comparison of observed and fitted time series is difficult because it is hard to assess the statistical significance. Instead, we analyze cross-correlograms. Figure 4 shows the sample cross-correlations of some key quantities with consumption growth, both in the model and the data. We focus on lead and lag correlations up to four quarters and also report the standard error bands.

The cross-correlation of inflation and consumption growth is not precisely measured as indicated by the wide standard error bands. Indeed, it is well-documented in the literature, such as in [Campbell, Sunderam, and Viceira \(2017\)](#), that the signs of these correlations flip over time. Arguably, our model provides a sensible account of the dynamic historical process. The estimated lead-lag correlations from the model for inflation, long-term yields, and debt mostly lie within the standard error bands of the empirical correlations, and often coincide in terms of sign. Given our emphasis on the fiscal conditions, it is reassuring that the model captures the dynamic behavior of debt especially well.

To address the imprecision associated with consumption-based cross-correlations, we display cross-correlogram of the same variables with the government expenditures, g_t , in Figure 5. Overall, our model provides a largely statistically significant account of the main joint dynamics, including those of CDS spreads and government expenditures at various lags. The theoretical magnitudes of correlations, however, overstate those in the data with the exception of CDS premiums.

The statistical properties of the time series behavior of CDS spreads in the data should at best viewed as suggestive. Indeed, limited data availability is reflected in the very wide standard error bands around the point estimates. Given the nature of

these estimates, our model implications have to be interpreted with some caution. However, our point estimates from the model are broadly in line with the ranges obtained from the data.

4.4 Counterfactuals

In the previous section, we evaluated the potential of our model to rationalize the observed CDS premiums based on fiscal default in the recently witnessed environment with high debt levels. We next address two questions via counterfactual analysis. First, which levels of CDS premiums and default probabilities would have been manifest in more tranquil fiscal conditions such as those prevailing for most of the postwar sample between 1947 and 2016? Second, conditional on reaching a high level of debt, can government policies be effective in reducing fiscal stress and reverting to the long-run debt levels previously observed, and what are the effects of such measures on CDS premiums? In this regard, we focus on the commonly discussed policies of taxing or inflating away debt.

4.4.1 Long-Run CDS Premiums

Given the lack of traded CDS contracts before 2007, we can only impute the default probabilities and hypothetical CDS premiums corresponding to periods of more moderate fiscal conditions through the lens of our model. Specifically, as discussed in section 4.2, we offer a counterfactual analysis of the postwar sample between 1947 and 2016. Table 6 displays the results.

By starting our simulations in a more moderate region of the state space, we can provide a reasonable account of the macroeconomic, monetary, and fiscal environment in the postwar era. Other than highly volatile tax rates and slightly low inflation, the approach well describes the dynamics of debt, taxes, and inflation. In particular, it closely matches the notably lower debt-to-GDP ratio in the U.S., which remained at around 60 percent for most of the postwar episode.

Naturally, given lower average debt levels, the likelihood of reaching states of severe fiscal stress is significantly lower. Accordingly, our model predicts somewhat minuscule average default probabilities for the fiscal conditions that prevailed for most of the postwar sample. Nevertheless, the economy supports a similar debt limit and maximum sustainable tax rate to that observed in the baseline scenario. No matter where the economy starts, it cannot go beyond certain limits.

Table 7 documents the pricing of fixed income securities and derivatives. Clearly, in more moderate fiscal conditions, the model predicts significantly muted CDS premiums. The differences from Table 5 are primarily driven by the default probability rather than the default risk premium. Indeed, the ratios of the CDS premiums to their risk-neutral counterparts, as measured by $L \cdot P_t^D$, continue to range between 2 and 3 in the long sample.

Regarding the term structure of government bonds, our model predicts somewhat lower nominal yields than what is observed in the data because of the model implied inflation dynamics. Although a Taylor rule appears to provide a good description of the monetary policy operations since the mid-eighties, it cannot account for the monetarist policies in the earlier stages of the postwar period. In particular, the model is not designed to give a quantitative account of the inflationary pressure

witnessed in the seventies, or the Volcker disinflation at the beginning of the eighties. By missing these episodes, the model understates both the average yields and average inflation.

4.4.2 Inflating and Taxing Away Debt

A common view is that a U.S. default is unlikely because the government can always resort to increasing taxation or creating inflation to restore the budget balance. We now examine a potential effect of such scenarios through the lens of our model. We represent an attempt to inflate away the debt burden with a shift toward a looser monetary policy stance. This is captured by a shift toward lower values of δ_π in the Taylor rule. Similarly, we can represent a shift toward a fiscal policy with more aggressive taxation by lowering ρ_b . This means that new debt is issued in smaller amounts, which would imply higher taxes via the GBC.

We now quantitatively evaluate the variation in the policies using counterfactual analysis. Table 8 reports the results for the monetary policy. Loosening the monetary policy stance has the desired effect of increasing the average inflation rate. Similarly, as expected, the average debt is reduced, which comes with a reduction in the default probability.

Remarkably, the CDS premiums rise. This happens because an increase in mean inflation is accompanied by an increase in its volatility. Because larger shocks to inflation make the fiscal limit more volatile, it can fall more relative to its mean, as highlighted in Figure 3. This decrease in the fiscal limit is accompanied by an increase in the default probability even though its mean declines. As a result, the risk premium amplification mechanism that we discussed in the previous section delivers

larger risk premiums despite the decline in the expected losses. It is worth noting that quantitatively the corresponding change in CDS premiums is small.

We obtain a similar result in the case of the attempt to “tax away” debt. As Table 9 shows, using taxes more aggressively to respond to economic conditions leads to a fall in the average debt burden and default probabilities, while the average tax rate increases. However, the volatility of taxes also increases, and, again according to Figure 3, the fiscal limit declines relative to its mean. As a result, the same mechanism is at play.

Our counterfactual exercises illustrate some of the pitfalls associated with attempts to inflate or tax away government debt obligations. Although these policies tend to have the desired effects for the first moments of debt, taxes, and inflation, they come with endogenous movements in the second moments. These movements are priced in our risk-sensitive framework and push the CDS risk premiums in the opposite direction.

5 Conclusion

The premiums on U.S. sovereign CDS rose to unprecedented levels during the recent financial crisis, and remain at elevated levels today. Given the apparent size of these premiums, commentators have widely speculated whether they indeed reflect the financial market expectations about an impending U.S. default. After all, casual inspection suggests that the U.S. government can always balance the budget by raising taxes or by inflating away the real value of debt. In this paper, we examine

whether the likelihood of a fiscal default, namely a state in which tax- or inflation-based finance is no longer available, justifies the size of the observed premiums.

We develop an equilibrium model of the U.S. economy with a representative agent featuring recursive preferences, in which the monetary and fiscal policies jointly endogenously determine the dynamics of growth, debt, taxes, and inflation. Fiscal default occurs when the economy approaches the slippery slope of the Laffer curve, a point at which a further increase in the tax rate will reduce the tax revenue. Our equilibrium approach allows us to value CDS contracts to reflect the risk-adjusted probabilities of fiscal default, thereby overcoming the challenge that the standard replication arguments for CDS pricing fail in the absence of the risk-free benchmark.

We find that our model quantitatively generates premiums on CDS contracts in line with those observed in the U.S. economy since the recent financial crisis. The annualized CDS premiums peak at around 100 bps in the model because the periods of high debt and default probability endogenously correspond to the periods of high marginal utility in the model, so that selling default insurance earns high risk premiums. Importantly, the CDS premiums rise persistently even in response to small shocks to the likelihood of fiscal default, because investors with recursive preferences anticipate and dislike such states. Our model is thus consistent with the view that high CDS premiums reflect investors' rational forecasts of the likelihood of U.S. fiscal stress.

Our results also cast a doubt on the idea that the government can restore the budget balance by simply inflating or taxing away debt. In the context of our model, although increased mean inflation and taxes are correlated with reduced average debt and default probabilities, they also bring about endogenous movements in the second

moments of these variables, both in terms of volatility and correlations. Although our partial equilibrium model merely suggests that such policies can also lead to increased risk premiums, such movements would likely have welfare implications in a richer general equilibrium framework.

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A Computational Procedure

The model is summarized by a system of expectational difference equations. Solving for the endogenous variables in the system is complicated by (i) the nonlinearities induced by the pricing kernel and the possibility of default and (ii) the endogeneity of the default boundary τ^* , which depends on present values of endogenous variables. We deal with (i) by adopting a global, nonlinear solution method based on projection techniques, and with (ii) by implementing an iterative algorithm based on Monte Carlo methods.

Our solution strategy to deal with (i) is to approximate the endogenous variables π_t, q_t^s , and q_t^l with flexible Chebyshev polynomials in the state variables $\varsigma_t = \{\tau_{t-1}, b_{t-1}, g_{t-1}, x_{t-1}, \varepsilon_t, \xi_t^b, \xi_t^q\}$. This amounts to solving for the coefficients of these polynomials that satisfy the model equations at specific points, namely the Chebyshev nodes. To find those, we choose bounds on the state variables and map those linearly into $[-1, 1]$, the domain of the Chebyshev polynomials. The bounds on the persistent stochastic variables x_t and g_t come from the [Tauchen \(1986\)](#) procedure to approximate an AR(1) process. Finding the coefficients of the Chebyshev polynomials at the relevant nodes thus translates into solving a nonlinear system of equations. To aid convergence, we start with lower order polynomials and successively increase the number of nodes.

To find the default boundary, we proceed as follows. For some initial default boundary $\tau_t^{*(0)} \equiv \tau^{*(0)}(\varsigma_t)$, we obtain the corresponding bond prices and inflation $q_t^{s,(0)}, q_t^{l,(0)}$, and $\pi_t^{(0)}$ using the Chebyshev collocation method described above. With these solutions at hand, we evaluate the expected sustainable (log) surplus s_t^* in any state ς_t via Monte Carlo simulations. Starting from any state ς_t , we simulate the model

forward for T periods to obtain s_t^* and an updated $\tau_t^{*(1)}$ for that state. Note that this $\tau^{*(1)}(\varsigma_t)$ depends on the endogenous variables $q_t^{s,(0)}$, $q_t^{l,(0)}$, and $\pi_t^{(0)}$, which were obtained as functions of the initial default boundary $\tau_t^{*(0)}$. We choose T sufficiently large to accommodate the persistence of the underlying processes.

Our algorithm then iterates back and forth between the projection step and the simulation step. More precisely, starting from any $\tau_t^{*(j)}$, we obtain an updated default boundary $\tau_t^{*(j+0.5)}$ by solving the model with projection and simulating it forward. Our aim is to iterate that procedure to convergence, so that $\max_{\varsigma_t} \|\tau^{*(j)}(\varsigma_t) - \tau^{*(j+0.5)}(\varsigma_t)\| < \bar{\varepsilon}$. To facilitate convergence, we implement a relaxation scheme by introducing $\tau^{*(j+1)}(\varsigma_t) = (1 - \zeta)\tau^{*(j)}(\varsigma_t) + \zeta\tau^{*(j+0.5)}(\varsigma_t)$, where ζ is a relaxation parameter. The convergence criterion becomes $\max_{\varsigma_t} \|\tau^{*(j)}(\varsigma_t) - \tau^{*(j+1)}(\varsigma_t)\| < \bar{\varepsilon}$. We also check that bond prices and inflation stabilize in the iterative process.

With the default boundary τ_t^* at hand, it is straightforward to evaluate the CDS premiums CDS_t^T on the grids. Our model statistics are computed from 100 simulations of 15 years of data, to be consistent with our empirical targets.

Table 1
Calibration

| Parameter | Description | Value |
|--|------------------------------------|-------------------------|
| <i>1. Preferences</i> | | |
| β | Subjective discount factor | 0.996 |
| ρ | IES= $(1 - \rho)^{-1}$ | 1/3 |
| α | RRA= $(1 - \alpha)$ | -9 |
| <i>2. Exogenous processes</i> | | |
| σ_c | Volatility of shocks to Δc | 0.014 |
| σ_x | Volatility of LLR process | $\sigma_c \times 0.044$ |
| φ_x | Autocorrelation of LLR | 0.936 |
| σ_y | Volatility of shocks to Δy | 0.022 |
| φ_y | Elasticity output growth - taxes | -0.024 |
| σ_g | Volatility of G | 0.075 |
| φ_g | Autocorrelation of G | 0.990 |
| <i>3. Different Maturities and Default</i> | | |
| ω | Share of short term debt | 0.200 |
| λ | Repayment rate | 0.040 |
| γ | Coupon payment | 0.010 |
| L | Losses in the default event | 0.300 |
| <i>4. Policy parameters</i> | | |
| δ_π | Inflation loading coefficient | 1.500 |
| δ_y | Output growth coefficients | 0.500 |
| ρ_b | Debt loading coefficient | 0.960 |
| ρ_x | Expected growth coefficient | -0.220 |

Notes. We describe the calibration in section [4.2](#).

Table 2
Macroeconomic Moments

| | Data | | Model | |
|------------------------------------|-------|-------|-------|-------|
| | Mean | Std | Mean | Std |
| Market value of debt ($Q_t B_t$) | 0.887 | 0.146 | 0.902 | 0.331 |
| Taxes (\mathcal{T}_t) | 0.326 | 0.031 | 0.349 | 0.163 |
| Annual gross inflation (Π_t) | 1.018 | 0.026 | 1.014 | 0.057 |
| Debt limit (S_t^*) | | | 1.511 | 0.119 |
| Tax limit (\mathcal{T}_t^*) | | | 0.843 | 0.064 |
| Default probability (P_t^D) | | | 0.001 | 0.002 |

Notes. This table reports basic macro moments. The empirical moments come from the BEA quarterly data and cover the sample period 2007:Q1 to 2016:Q2. All moments are annualized. To compute theoretical moments, we simulate the data at quarterly frequency 100 times for 10 years and average across simulations.

Table 3
Correlations

| | $\log(Q_t B_t)$ | τ_t | g_t | s_t^* | τ_t^* | $\log P_t^D$ | Π_t |
|--------------|-----------------|----------|--------|---------|------------|--------------|---------|
| Δc_t | -0.659 | -0.925 | -0.995 | 0.832 | 0.796 | -0.901 | -0.853 |

Notes. This table reports correlations between main variables in our model. We simulate the data at quarterly frequency 100 times for 10 years and average across simulations.

Table 4
Term Structure of Bond Yields

| Maturity, years | Model | | | Data |
|--------------------|-------|------------------|---------|---------|
| | Real | Pseudo risk-free | Nominal | Nominal |
| 1 | 0.43 | 1.55 | 1.67 | 0.91 |
| 3 | 0.33 | 2.09 | 2.25 | 1.42 |
| 5 | 0.22 | 2.42 | 2.59 | 2.08 |
| 10 | 0.13 | 3.08 | 3.27 | 2.96 |

Notes. This table reports annualized mean yields across horizons of various fixed income instruments in the model and the data. The empirical moments correspond to U.S. nominal Treasury bonds in the sample between 2007:Q1 to 2016:Q2. We simulate the data at quarterly frequency 100 times for 10 years and average across simulations.

Table 5
CDS Spreads

| Maturity, years | Data euro (1) | Data euro (2) | Data \$ (3) | Model (4) | $L \cdot \text{Mean}(P_t^D)$ (5) |
|--------------------|------------------|------------------|----------------|--------------|-------------------------------------|
| 1 | 16.13 | 15.61 | 13.29 | 8.84 | 3.14 |
| 3 | 22.18 | 21.66 | 17.32 | 11.73 | 4.73 |
| 5 | 29.34 | 31.29 | 23.11 | 15.61 | 6.09 |
| 10 | 40.79 | 46.86 | 34.92 | 24.31 | 10.13 |

Notes. This table reports annualized mean CDS premiums across maturities in the data and the model. Column (1) displays EUR-denominated contracts from 2007:Q1 to 2016:Q2. Column (3) displays USD-denominated contracts from 2010:Q3 to 2016:Q2. Column (2) shows EUR-denominated contracts for the same sample as the USD-denominated ones. Column (4) shows the theoretical premiums. The table also reports theoretical expected losses on the government debt portfolio in column (5). We simulate the data at quarterly frequency 100 times for 10 years and average across simulations.

Table 6
Long-Run Macroeconomic Moments

| | Data | | Model | |
|------------------------------------|-------|-------|--------|-------|
| | Mean | Std | Mean | Std |
| Market value of debt ($Q_t B_t$) | 0.573 | 0.219 | 0.586 | 0.235 |
| Taxes (\mathcal{T}_t) | 0.302 | 0.053 | 0.317 | 0.148 |
| Annual gross inflation (Π_t) | 1.033 | 0.032 | 1.012 | 0.047 |
| Debt limit (S_t^*) | | | 1.717 | 0.079 |
| Tax limit (\mathcal{T}_t^*) | | | 0.876 | 0.042 |
| Default probability (P_t^D) | | | 0.0005 | 0.001 |

Notes. This table reports basic macro moments. The empirical moments come from the BEA quarterly data and cover the sample period 1947:Q1 to 2016:Q2. All moments are annualized. To compute theoretical moments, we simulate the data at quarterly frequency 100 times for 70 years and average across simulations.

Table 7
Long-Run Yields and Premiums

| Maturity, years | Model, Default | | Real | Model, Yields | | Data Nominal |
|--------------------|----------------|------------------------------|------|------------------|---------|-----------------|
| | CDS | $L \cdot \text{Mean}(P_t^D)$ | | Pseudo risk-free | Nominal | |
| 1 | 3.81 | 1.53 | 0.45 | 1.61 | 1.69 | 5.14 |
| 3 | 5.76 | 2.82 | 0.35 | 2.10 | 2.21 | 5.72 |
| 5 | 9.54 | 4.19 | 0.22 | 2.49 | 2.62 | 6.13 |
| 10 | 16.65 | 7.03 | 0.13 | 3.58 | 3.69 | 6.42 |

Notes. This table reports annualized mean yields across horizons of various fixed income instruments in the model and the data. The empirical moments correspond to U.S. nominal Treasury bonds in the sample between 1947:Q1 to 2016:Q2. We simulate the data at quarterly frequency 100 times for 70 years and average across simulations.

Table 8
Monetary Policy and CDS Premiums

| δ_π | $Q_t B_t$ | Mean | | Π_t | Std |
|--------------|-----------|------------------------------|---------|---------|---------|
| | | $L \cdot \text{Mean}(P_t^D)$ | CDS_5 | | Π_t |
| 1.50 | 0.902 | 6.09 | 15.61 | 1.014 | 0.057 |
| 1.45 | 0.891 | 6.01 | 16.12 | 1.019 | 0.064 |
| 1.40 | 0.865 | 5.89 | 16.33 | 1.022 | 0.069 |

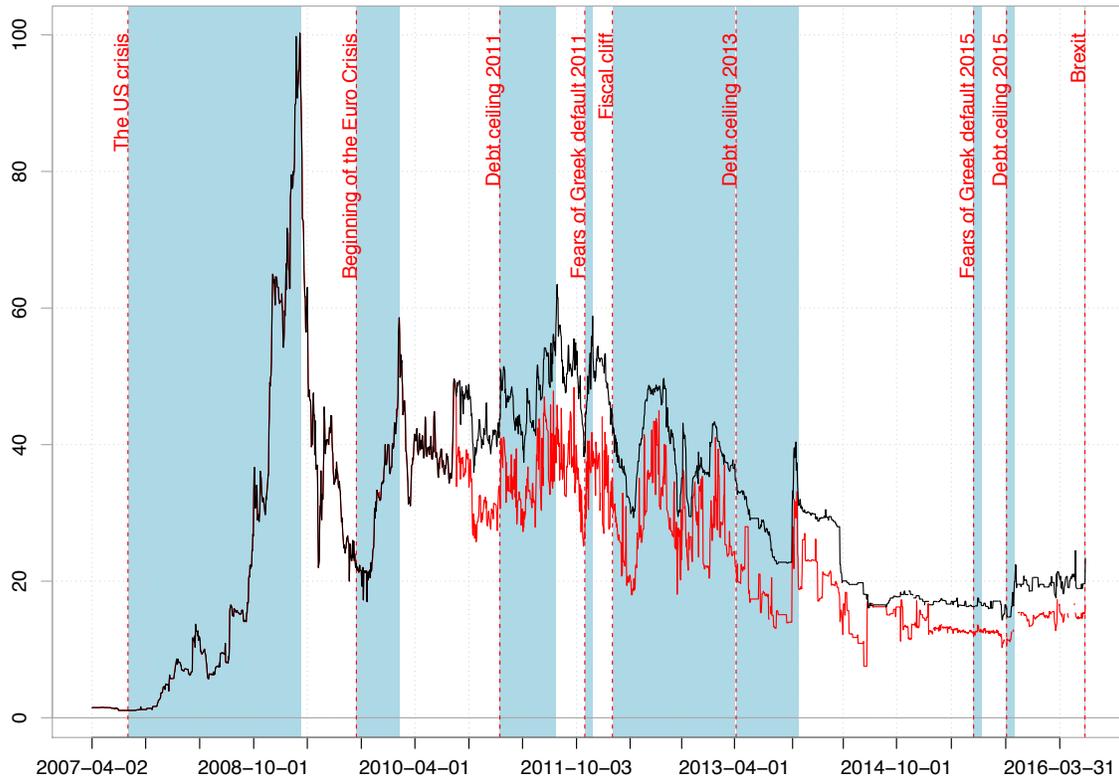
Notes. This table reports theoretical moments corresponding to various magnitudes of monetary policy response to inflation, δ_π . Means and standard deviations are annualized. We simulate the data at quarterly frequency 100 times for 10 years and average across simulations.

Table 9
Fiscal Policy and CDS Premiums

| ρ_b | $Q_t B_t$ | Mean | | \mathcal{T}_t | Std |
|----------|-----------|------------------------------|------------|-----------------|-----------------|
| | | $L \cdot \text{Mean}(P_t^D)$ | $CDS_t(5)$ | | \mathcal{T}_t |
| 0.98 | 0.916 | 6.26 | 15.44 | 0.341 | 0.152 |
| 0.96 | 0.902 | 6.09 | 15.61 | 0.349 | 0.163 |
| 0.94 | 0.885 | 5.98 | 15.74 | 0.356 | 0.174 |

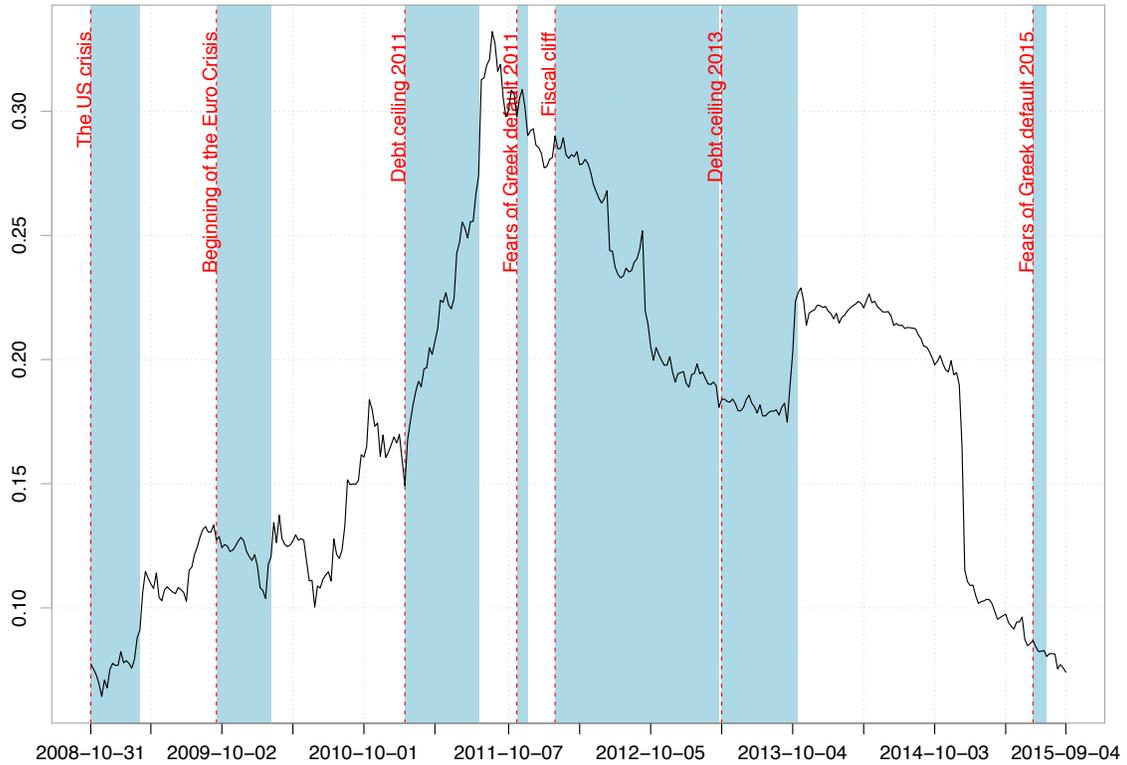
Notes. This table reports theoretical moments corresponding to various magnitudes of fiscal policy response to debt, ρ_b . Means and standard deviations are annualized. We simulate the data at quarterly frequency 100 times for 10 years and average across simulations.

Figure 1
History of U.S. CDS premiums



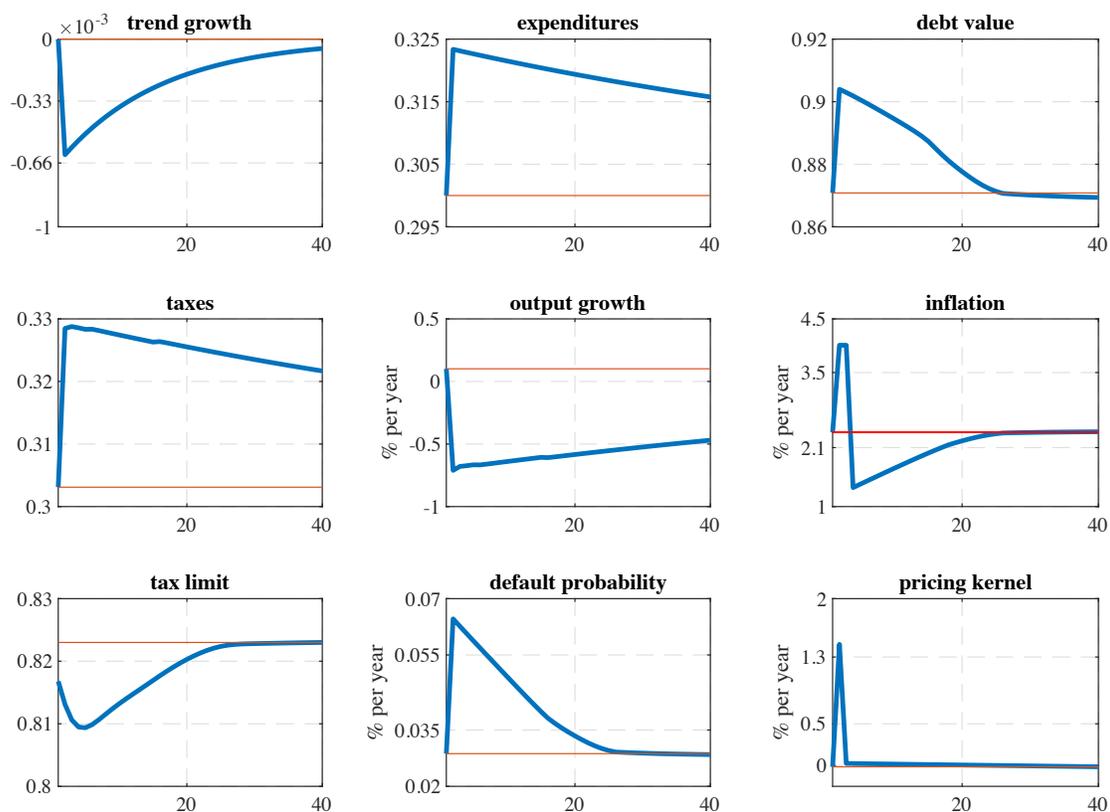
Notes. We plot the time-series of premiums on five-year contracts. The dark line represents quotes in EUR from April 2007 to June 2016, and the light one is in USD from August 2010 to June 2016. The time series are complemented by the highlights of major economic and political events during that period. The premiums are expressed in basis points per year.

Figure 2
Liquidity of U.S. CDS



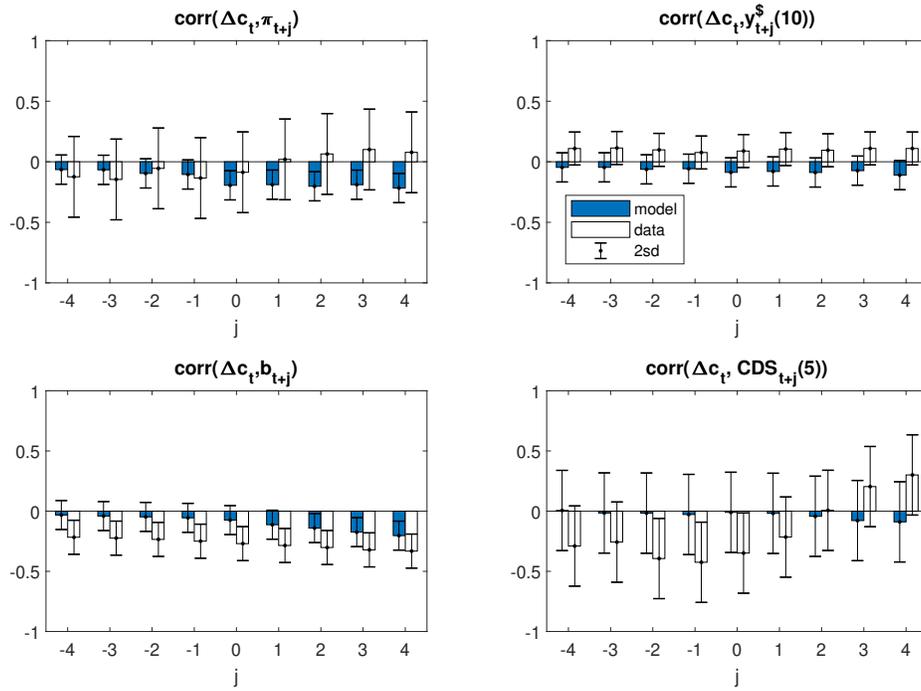
Notes. We plot the time-series of liquidity of the U.S. CDS market. CDS contracts on Italian government are the most actively traded sovereign contracts. For this reason, our liquidity measure is equal to the ratio of the weekly net notional amount of U.S. CDS to that of Italian CDS. The corresponding USD (in MM) levels have an average (median) of 3,195 (3,293) and min (max) of 1,152 (5,946). The time series is complemented by the highlights of major economic and political events.

Figure 3
Impulse Responses



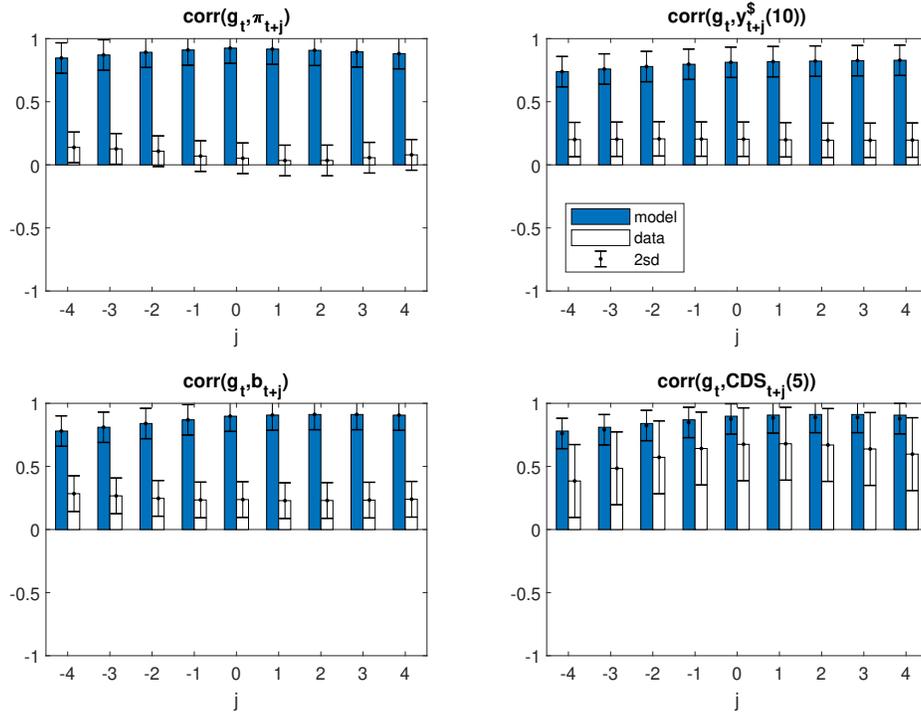
Notes. We plot the responses of model variables (bold, blue line) to a one standard deviation negative innovation ε_t to the long-run consumption trend, over 40 quarters. We display their steady state values (thin, red line) as well. Stock variables are expressed in levels. Flow rates are expressed in annualized percent.

Figure 4
Cross-Correlations with Consumption Growth



Notes. We plot annual cross-correlations between model variables and consumption growth at j lags and leads. Standard errors are indicated.

Figure 5
Cross-Correlations with Government Expenditures



Notes. We plot annual cross-correlations between model variables and government expenditures at j lags and leads. Standard errors are indicated.