CDS Auctions *

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Abstract We analyze auctions for the settlement of credit default swaps (CDS) theoretically and evaluate them empirically. The requirement to settle in cash with an option to settle physically leads to an unusual two-stage process. In the first stage, participants affect the amount of the bonds to be auctioned off in the second stage. Participants in the second stage may hold positions in derivatives on the assets being auctioned. We show that the final auction price might be either above or below the fair bond price, due to strategic bidding on the part of participants holding CDS. Empirically, we observe both types of outcomes, with undervaluation occurring in most cases. We find that auctions undervalue bonds by an average of 6% on the auction day. Undervaluation is related positively to the amount of bonds exchanged in the second stage of the auction, as predicted by theory. We suggest modifications of the settlement procedure to minimize the underpricing.

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Introduction

Credit Default Swaps (CDS) have been one of the most significant financial innovations in the last 20 years. They have become very popular among investment and commercial banks, insurance companies, pension fund managers and many other economic agents. As a result, the market has experienced enormous growth. According to the Bank of International Settlements (BIS), the notional amount of single-name CDS contracts grew from $5.1 trillion in December 2004 to $33.4 trillion in June 2008, and was still $16.8 trillion in December 2011, following a decline in the aftermath of the credit crisis.

The recent crisis put CDS in the spotlight, with policymakers now assigning them a central role in many reforms. The success of these reforms depends on the efficient functioning of the CDS market and on a thorough understanding of how it operates. Recognizing this, much research has been dedicated to the valuation of CDS contracts, econometric analysis of CDS premia, violations of the law of one price in the context of basis trades, search frictions, counterparty risk, private information, and moral hazard associated with holding both bonds issued by a particular entity and CDS protection on this entity.[1]

Herein, we study another aspect of CDS: how the payoff of a CDS contract is determined when a credit event occurs. Our theoretical analysis of the unusual auction-based procedure reveals that this mechanism may lead to deviations from the fair bond price. We attribute the mispricing to strategic bidding on the part of investors holding CDS. Empirically, we find that CDS auctions undervalue the underlying securities in most cases. At our most conservative estimates, the average underpricing is 6%. This cost is of the same order of magnitude as documented in the literature on Treasury auctions, and initial and seasoned equity offerings. Because this mispricing is large, our findings may have implications for how CDS are valued, used and analyzed.

A CDS is a contract that protects a buyer against the loss of a bond's principal in the case of a credit event (e.g., default, liquidation, and debt restructuring). Initially,

[1]This work includes, but is not limited to, Acharya and Johnson (2007), Arora, Gandhi, and Longstaff (2009), Bolton and Oehmke (2011), Duffie (1999), Duffie and Zhu (2011), Garleanu and Pedersen (2011), Longstaff, Mithal, and Neis (2005), Pan and Singleton (2008), and Parlour and Winton (2010).

CDS were settled physically. Under such settlement, the buyer of protection was required to deliver any bond issued by the reference entity to the seller of protection in exchange for the bond's par value. However, as a result of the rapid growth of the CDS market, the notional amount of outstanding CDS contracts came to exceed the notional amount of deliverable bonds many times over. This made physical settlement impractical and led the industry to develop a cash settlement mechanism. This mechanism is the object of our study.

While many derivatives are settled in cash, the settlement of CDS in this way is challenging for two reasons. First, the underlying bond market is opaque and illiquid, which makes establishing a benchmark bond price difficult. Second, parties with both CDS and bond positions face so-called recovery basis risk if their positions are not closed simultaneously.[2] The presence of this risk renders it necessary that the settlement procedure include an option to replicate an outcome of the physical settlement.

In response to these challenges, the industry has developed a novel two-stage auction. In the first stage, parties that wish to replicate the outcome of the physical settlement submit their requests for physical delivery via dealers. These requests are aggregated into the net open interest (NOI). Dealers also submit bid and offer prices with a commitment to transact in a predetermined minimal amount at the quoted prices. These quotations are used to construct the initial market midpoint price (IMM). The IMM is used to derive a limit on the final auction price, which is imposed to avoid manipulation of prices. The limit is referred to as the price cap. The NOI and the IMM are announced to all participants.

In the second stage, a uniform divisible good auction is implemented, in which the net open interest is cleared. Each participant may submit limit bids that are combined with the bids of the dealers from the first stage. The bid that clears the net open interest is declared to be the final auction price, which is then used to settle

[2] Recovery basis risk can be illustrated as follows. Suppose a party wishes to hedge a long position in a bond by buying a CDS with the same notional amount. The final physically-settled position is known in advance: the buyer of protection delivers a bond in exchange for a predetermined cash payment equal to par value. However, the cash-settled position is uncertain before the auction: the buyer of protection keeps the bond, pays the auction-determined bond value (unknown at the outset) to the seller of protection, and receives par value in exchange. The difference between the market value of the bond held by the buyer of protection and the auction-determined value is the recovery basis.

the CDS contracts in cash.

We analyze the auction outcomes from both theoretical and empirical perspectives. To study price formation, we follow Wilson (1979) and Back and Zender (1993). We formalize the auction using an idealized setup in which all auction participants are risk-neutral and have identical expected valuations of the bond, v. This case is not only tractable, but also provides a useful benchmark against which to test whether CDS auctions lead to the fair-value price. CDS auctions differ from standard ones in two critical respects.

First, the participants in the auction affect how many bonds are traded in the auction and in which direction (NOI) because of their ability to submit physical settlement requests in the first stage. Second, participants in the auction's second stage may have positions in derivatives on the asset being auctioned.

Taking the NOI and CDS positions at the second stage as given, we demonstrate that the second stage of the auction can result in a final price either above or below v. This conclusion differs from the case of a standard uniform divisible good auction that can result in underpricing only. To be specific, consider the case of positive NOI: a second-stage auction in which the agents buy bonds. Note that sellers of protection benefit if the auction price is set above the fair value. When the aggregate CDS positions of sellers of protection are less than the NOI, the Wilson (1979) argument still holds. Underpricing occurs if the participants choose not to bid aggressively. The current auction rule is such that bids above the final price are guaranteed to be fully filled, so participants are not sufficiently rewarded for raising their bids. On the other hand, when the aggregate CDS positions of sellers are larger than the NOI, bidding above the fair value and realizing a loss from buying NOI units of bonds is counterbalanced by a reduction in the net payoff of the existing CDS contracts. As a result, the auction price can be greater than v in the absence of a cap.

The outcome of the first stage imposes restrictions on possible outcomes in the second stage. We distinguish two scenarios. First, we consider a case of no trading frictions. Then, continuing with the example of the positive NOI, we show that the NOI can never be greater than the aggregate CDS positions of sellers. Thus, underpricing equilibria are not possible. Moreover, participants in the auction are indifferent to the possible overpricing because they can undo any adverse effects of second-stage bidding on the value of their positions by choosing optimally between

physical and cash settlement.

Then, we consider a more realistic setup where there are constraints on short selling, and where some participants cannot hold distressed debt. We prove that both underpricing and overpricing are possible because the aggregate CDS positions of sellers who participate in the auction can be either smaller or larger than the NOI. Moreover, we show that underpricing should be an increasing function of the NOI for the underpricing equilibria to be realized in a two-stage auction. Simultaneously, participants in the auction are no longer indifferent to auction outcomes because they might not be able to implement their optimal choice between physical and cash settlement.

Our theory delivers a rich set of testable predictions. The testing of every prediction requires data on individual CDS positions and bids, which are not available. Nonetheless, we are able to analyse some aspects of the auction data and find evidence that is consistent with our theoretical predictions. We use TRACE bond data to construct the reference bond price. Using this price as a proxy for v, we find that the auction price is set at the price cap whenever there is overpricing. When the final auction price is uncapped and the NOI is positive (a typical situation), the bonds are undervalued and the degree of undervaluation increases with the NOI. In addition, underlying bond prices follow a V pattern around the auction day. In the 10 days before the auction, prices decrease by 25% on average. They reach their lowest level on the day of the auction (average underpricing of 6%), before reverting to their pre-auction levels over the next 10 days. This evidence suggests that our conclusions on the prevailing underpricing are robust to the choice of the reference bond price.

Our findings prompt us to consider ways to mitigate the observed mispricing. Kremer and Nyborg (2004b) suggest a likely source of underpricing equilibria in a standard setting, in which agents have no prior positions in derivative contracts written on the asset being auctioned. They show that a simple change of allocation rule from pro-rata on the margin to pro-rata destroys all underpricing equilibria. We show that the same change of allocation rule would be beneficial in our setting. In addition, we suggest that imposing a price cap on the auction that is conditional on the outcome of the first stage could further reduce mispricing in equilibrium outcomes.

To our knowledge, there are four other papers that examine CDS auctions. Helwege, Maurer, Sarkar, and Wang (2009) find no evidence of mispricing in an early

sample of 10 auctions. However, only four used the current auction format. Coudert and Gex (2010) study a somewhat different sample of auctions, using Bloomberg data for reference bond prices. They document a large gap between a bond's price on the auction date and the final auction price. However, they do not link the gap to the net open interest, nor do they provide any theoretical explanations for their findings. Gupta and Sundaram (2012) also report a V pattern in bond prices around the auction day. Under a simplifying assumption that bidders in the second stage of the auction have zero CDS positions, they find that a discriminatory auction format could reduce the mispricing. Finally, similarly to us, Du and Zhu (2012) examine the outcomes that are possible in CDS auctions theoretically. They neither solve for the optimal choice between physical and cash settlement nor consider the effect of trading frictions on equilibrium outcomes. As a result, they conclude that only overpricing equilibria can exist and propose double-auction format to mitigate this type of outcomes.

The remainder of the paper is organized as follows. Section 1 describes current methods of conducting CDS auctions. Section 2 describes the auction model. Section 3 provides the main theoretical analysis. Section 4 relates the predictions of the theoretical model to empirical data from CDS auctions. Section 5 discusses modifications that might improve the efficiency of the auction. Section 6 concludes. The appendix contains proofs that are not provided in the main text.

1 The Auction Format

This discussion is based on a reading of the auction protocols available from the ISDA website. The first single-name auction that followed the current format was conducted on November 28, 2006 for the Dura credit event. The auction design used in this case, and for all subsequent credit events, consists of two stages.

In the first stage, participants in the auction submit their requests for physical settlement. Each request for physical settlement is an order to buy or sell bonds at the auction price. To the best of the relevant party's knowledge, the order must be in the same direction as – and not in excess of – the party's market position, which allows the participants to replicate traditional physical settlement of the contracts.

In addition, a designated group of agents (dealers) makes a two-way market in the

bonds of defaulted entities by submitting bids and offers with a predefined maximum bid-ask spread and associated quotation size. The bid-ask spread and quotation sizes are stipulated in the auction protocol and may vary depending on the liquidity of the defaulted assets.[3]

The inputs to the first stage are then used to calculate the net open interest (NOI) and the initial market midpoint (IMM), which are carried through to the second stage. The NOI is computed as the difference of the physical-settlement buy and sell requests. The IMM is set by discarding crossing/touching bids and offers, taking the 'best half' of each, and calculating the average. The best halves would be, respectively, the highest bids and the lowest offers. If a dealer's quotation is crossed and if either (a) her bid is higher than the IMM and the NOI is to sell, or (b) an offer is lower than the IMM and the NOI is to buy, she must make a payment, called an adjustment amount, to the ISDA. The adjustment amount is a product of the quotation amount and the difference between the quotation and the IMM.

After the publication of the IMM, the NOI, and the adjustment amounts, the second stage of the auction begins. If the NOI is zero, the final price is set equal to the IMM. If the NOI is non-zero, dealers may submit corresponding limit orders on behalf of their clients (including those without CDS positions) – and for their own account – to offset the NOI. Agents are allowed to submit 'buy' limit orders only if the NOI is greater than zero and 'sell' limit orders only if it is less than zero.

Upon submission of the limit orders, if the NOI is to buy, the auction administrators match the open interest against the market bids from the first stage of the auction and against the limit bids from the second stage. They start with the highest bid, proceeding through the second highest bid, third highest bid, and so on, until either the entire net open interest or all of the bids have been matched. If the NOI is cleared, the final price is set equal to the lowest bid corresponding to the last-matched limit order. However, if this bid exceeds the IMM by more than a pre-specified spread (typically, half of the bid-ask spread), the final price is simply set equal to the IMM plus the spread. If all bids are matched before the NOI clears, the final price will be zero and all bids will be filled on a pro-rata basis. The procedure is similar if the NOI is to sell, in particular, the final price has a floor equal to the

[3]The most common value of the bid-ask spread is 2% of par. Quotation sizes range from \$2 to \$10 million; \$2 million is the most common amount.

IMM minus the spread. If there are not enough offers to match the net open interest, the final price is set to par.

As an example, consider the Washington Mutual auction of October 23, 2008. Table 1 lists the market quotations submitted by participating dealers. Once these quotations have been received, the bids are sorted in descending order and the offers in ascending order. The highest bid is then matched with the lowest offer, the second highest bid with the second lowest offer, and so on. Figure 1 displays the quotations from Table 1, which are organized in this way. For example, the Dresdner Bank AG bid of 64.25 and the Credit Suisse International offer of 63.125 create a tradeable market.

The IMM is computed from the non-tradeable quotations. First, the 'best half' of the non-tradeable quotations is selected (i.e., the first five pairs). Second, the IMM is computed as an average of bid and offer quotations in the best half, rounded to the nearest one-eighth of a percentage point. In our example, there are six pairs of such quotations. The relevant bids are 63.5, four times 63 and 62.5. The relevant offers are two times 64; three times 64.25; and 64.5. The average is 63.6 and the one rounded to the nearest eights is 63.625.

Given the established IMM and the direction of open interest, dealers whose quotations have resulted in tradeable markets pay the adjustment amount to the ISDA. In the case of Washington Mutual, the open interest was to sell. Thus, dealers whose bids crossed the markets were required to pay an amount equal to (Bid-IMM) times the quotation amount, which was \$2 million. Dresdner Bank AG had to pay $(64.25 - 63.625)/100 \times \$2MM = \$62500$.

Finally, the direction of open interest determines the cap on the final price, where the price itself is set in the second part of the auction. In the Washington Mutual example, the open interest was to sell and the spread was 1.0, which meant the final price could exceed the IMM by a maximum of 1.0. Thus, the price cap was 64.625, as depicted in Figure 1.

2 The Auction Model

The main issue we address herein is whether the current auction format can result in mispricing. Our approach is motivated by the classic work of Wilson (1979) and

Back and Zender (1993), who show how this can happen in a standard divisible-good auction. As in Wilson (1979) and Back and Zender (1993), we assume that all agents are risk-neutral and have identical expectations about the value of the bonds. This case is not only tractable, but also provides a useful benchmark against which to judge whether the auction leads to the fair-value price.

The goal of this section is to formalize the auction process described in Section 1. There are two dates: $t = 0$ and $t = 1$. There is a set $\mathcal{N}$ of strategic players. A set of dealers $\mathcal{N}_d$ constitutes a subset of all players, $\mathcal{N}_d \subseteq \mathcal{N}$. Each agent $i \in \mathcal{N}$ is endowed with $n_i \in \mathbb{R}$ units of CDS contracts and $b_i \in \mathbb{R}$ units of bonds. Agents with positive (negative) n_i are called protection buyers (sellers). Because a CDS is a derivative contract, it is in zero net supply, $\sum_i n_i = 0$. One unit of bond pays $\tilde{v} \in [0, 100]$ at time $t = 1$. The auction starts at time $t = 0$ and consists of two stages.

2.1 First Stage

In the first stage, the auction IMM and NOI are determined. Agent i may submit a request to sell y_i (or buy if $y_i < 0$) units of bonds at the auction price p^A. Each protection buyer, $n_i > 0$, is only allowed to submit a request to sell $y_i \in [0, n_i]$ units of bonds, while each protection seller, $n_i < 0$, may only submit a request to buy $y_i \in [n_i, 0]$ units of bonds. Given these requests, the NOI is determined as follows:

$$NOI = \sum_{i \in \mathcal{N}} y_i. \tag{1}$$

In addition, all dealers from the set $\mathcal{N}_d$ are asked to provide a quotation for the price π_i. Given π_i, dealer i must stand ready to sell or buy L units of bonds at bid and offer prices $\pi_i + s$ and $\pi_i - s$, $s > 0$. Quotations from dealers whose bids and offers cross are discarded. The IMM is then set equal to the average of the remaining mid-quotations.

2.2 Second Stage

In this stage, a uniform divisible good auction is held. If $NOI = 0$, $p^A = IMM$. If $NOI > 0$, participants bid to buy NOI units of bonds. In this case, each agent $i \in \mathcal{N}$ may submit a left-continuous nonincreasing demand schedule $x_i(p) : [0, IMM + s] \rightarrow$

$\mathbb{R}_+ \cup 0$ via a dealer from the set $\mathcal{N}_d$. Let $X(p) = \sum_{i \in \mathcal{N}} x_i(p)$ be the total demand. The final auction price p^A is the highest price at which the entire NOI can be matched:

$$p^A = \max\{p | X(p) \geq NOI\}.$$

If $X(0) \leq NOI$, $p^A = 0$. Given p^A, the allocations $q_i(p^A)$ are determined according to the 'pro-rata at the margin' rule:

$$q_i(p^A) = x_i^+(p^A) + \frac{x_i(p^A) - x_i^+(p^A)}{X(p^A) - X^+(p^A)} \times (NOI - X^+(p^A)), \tag{2}$$

where $x_i^+(p^A) = \lim_{p \downarrow p^A} x_i(p)$ and $X^+(p) = \lim_{p \downarrow p^A} X(p)$ are the individual and total demands, respectively, above the auction clearing price.

If $NOI < 0$, participants offer to sell $|NOI|$ units of bonds. Each agent $i \in \mathcal{N}$ may then submit a right-continuous nondecreasing supply schedule $x_i(p) : [100, \max\{IMM - s, 0\}] \rightarrow \mathbb{R}_- \cup 0$ via a dealer from the set $\mathcal{N}_d$.

As before, the total supply is $X(p) = \sum_{i \in \mathcal{N}} x_i(p)$. The final auction price p^A is the lowest price at which the entire NOI can be matched:

$$p^A = \min\{p | X(p) \leq NOI\}.$$

If $X(100) \geq NOI$, $p^A = 100$. Given p^A, the allocations $q_i(p^A)$ are given by

$$q_i(p^A) = x_i^-(p^A) + \frac{x_i(p^A) - x_i^-(p^A)}{X(p^A) - X^-(p^A)} \times (NOI - X^-(p^A)),$$

where $x_i^-(p^A) = \lim_{p \uparrow p^A} x_i(p)$ and $X^-(p) = \lim_{p \uparrow p^A} X(p)$ are the individual and total supplies, respectively, below the auction clearing price.

2.3 Preferences

We assume that all agents are risk-neutral and have identical expected valuations of the bond payoff, v. The agents' objective is to maximize their wealth, Π_i, at date 1,

where

$$
\begin{aligned}
\Pi_i \quad = \quad & \underbrace{q_i(v - p^A)}_{\text{auction-allocated bonds}} + \underbrace{(n_i - y_i)(100 - p^A)}_{\text{net CDS position}} \\
+ \quad & \underbrace{y_i(100 - v)}_{\text{physical settlement}} + \underbrace{b_i v}_{\text{initial bonds}}.
\end{aligned} \tag{3}
$$

and q_i is the number of auction-allocated bonds. The net CDS position is a position that remains after participants submit their requests for physical settlement.

2.4 Trading Constraints

So far, we have assumed a frictionless world in which every agent can buy and sell bonds freely. This is a very strong assumption; it is violated in practice. Therefore, we extend our setup to allow market imperfections. Specifically, we place importance on the following two frictions.

First, because bonds are traded in OTC markets, short-selling a bond is generally difficult. To model this, we introduce Assumption 1:

Assumption 1 *Each agent i can sell only b_i units of bonds.*

Second, some auction participants, such as pension funds or insurance companies, may not be allowed to hold bonds of defaulted companies. To model this, we introduce Assumption 2:

Assumption 2 *Only a subset $\mathcal{N}_+ \subseteq \mathcal{N}$, $\mathcal{N}_+ \neq \emptyset$ of the set of agents can hold a positive amount of bonds after the auction.*

In what follows, we solve for the auction outcomes both in the frictionless world and under Assumptions 1 and 2.

3 Analysis

In this section, we provide a formal analysis of the auction described in the preceding section. We begin by assuming that the second stage of the auction does not have a cap. After we solve for (and develop intuition about) the auction outcomes, we discuss the effect of the cap. We solve for the auction outcomes using backward induction.

We start by solving for the equilibrium outcome in the second stage of the auction, for a given NOI. We then find optimal physical settlement requests in the first stage, given the equilibrium outcomes of the second stage.

3.1 Second Stage

As noted above, the second stage consists of a uniform divisible good auction with the goal of clearing the net open interest generated in the first stage. A novel feature of our analysis is that we study auctions where participants have prior positions in derivative contracts written on the asset being auctioned. We show that equilibrium outcomes in this case can be very different from those realized in 'standard' auctions (that is, auctions in which $n_i = 0$ for all i).

We first consider the case in which all CDS positions are common knowledge. This assumption is relaxed in Section 5.4. If this is the case, each agent i takes the following as given: the NOI, a set of all CDS positions n_i, a set of physical settlement requests y_i, $i \in \mathcal{N}$, and the demand of other agents $x_{-i}(p)$. Therefore, from equation (3), each agent's demand schedule $x_i(p)$ solves the following optimization problem:

$$\max_{x_i(p)} \left(v - p^A(x_i(p), x_{-i}(p))\right) q_i(x_i(p), x_{-i}(p)) + (n_i - y_i)\left(100 - p^A(x_i(p), x_{-i}(p))\right). \quad (4)$$

The first term in this expression represents the payoff realized by participating in the auction, while the second term accounts for the payoff from the remaining CDS positions, $n_i - y_i$, which are settled in cash on the basis of the auction results.

Without CDS positions, (4) is the standard auction setup that is studied in Wilson (1979) and Back and Zender (1993). Wilson (1979) and Back and Zender (1993) show that the auction can result in a price that is below v. Underpricing can occur if the participants in the auction choose not to bid aggressively. Due to the fact that bids above the final price are guaranteed to be fully filled, participants are not, in general, rewarded sufficiently for raising their bids.

With CDS positions, holding the payoff from the auction constant, an agent who has a short (long) remaining CDS position wants the final price to be as high (low) as possible. However, agents with opposing CDS positions do not have the same capacity to affect the auction price. The auction design restricts participants to submitting one-sided limit orders, depending on the sign of the NOI.

If the $NOI > 0$, only buy limit orders are allowed. Therefore, all that an agent with a long CDS positions can do to promote her desired outcome is not to bid at all. By contrast, agents with short CDS positions are capable of bidding up the price. The situation is reversed when the $NOI < 0$. In what follows, we focus on the case of $NOI > 0$, as the most empirically relevant. The results for the case of $NOI < 0$ are parallel and are available upon request.

To illustrate the case, consider an example of one agent with a short CDS position. She has an incentive to bid the price as high as possible if the NOI is lower than the notional amount of her CDS contracts. This is because the cost of purchasing the bonds at a high auction price is offset by the benefit of cash-settling her CDS position at the same high price. In contrast, if the NOI is larger than the notional amount of her CDS position, she would not want to bid more than the fair value of the bond, v. This is because the cost of purchasing bonds at a price above v is not offset by the benefit of cash-settling her CDS position.

Similar intuition holds when there is a subset of agents whose aggregate short net CDS positions are larger than the NOI. In this case, their joint loss that is incurred by acquiring a number of bonds equal to the NOI, at a price above v, is dominated by a joint gain from paying less on a larger number of short CDS contracts that remain after the physical settlement. As a result, these agents will bid aggressively and can push the auction price above v.

The above intuition is formalized in Proposition 1, which extends the Wilson (1979) and Back and Zender (1993) results to the case in which agents can have CDS positions.

Proposition 1 *Suppose that there is an auction to buy $NOI > 0$ units of bonds. Each participating agent i solves optimization problem (4). If*

$$\sum_{i:\, n_i - y_i < 0} |n_i - y_i| \geq NOI, \tag{5}$$

then the final auction price $p^A \in [v, 100]$ in any equilibrium. If the condition (5) does not hold, then the final auction price $p^A \in [0, v]$ in any equilibrium.

Proof. See the Appendix.

3.2 First Stage

In order to solve for a full-game equilibrium we have to determine optimal physical settlement requests y_i and the NOI, given the outcomes in the second stage of the auction. We consider a benchmark case without trading frictions in Section 3.2.1. In Section 3.2.2, we consider more realistic settings that allow for trading frictions.

3.2.1 Auction without Trading Frictions

We show that only equilibria of a special type exist. First, without trading frictions, in any equilibrium, the NOI can never be less (more) than net CDS positions of protection sellers (buyers) if the NOI is positive (negative). In other words, condition (5) always holds. From the definition of the NOI,

$$\sum_{i:n_i<0}(n_i-y_i)+NOI=\sum_{i:n_i<0}(n_i-y_i)+\sum_i y_i=\sum_{i:n_i<0}n_i+\sum_{i:n_i>0}y_i\leq\sum_{i:n_i<0}n_i+\sum_{i:n_i>0}n_i=0.$$

Therefore, according to Proposition 1, all mispricing equilibria are unidirectional; there is no under- (over-) pricing if the NOI is positive (negative). Second, regardless of the equilibrium outcome, all agents gain the same utility as they would if p^A were equal to v. The reason for this is that all agents can undo any loss of utility that might result from mispricing in the second stage by optimally choosing between cash and physical settlement of their positions in the first stage.

Proposition 2 *Suppose that there are no trading frictions, that is, Assumptions 1 and 2 are not imposed. Then in any equilibrium, one of the following three outcomes can be realized: (i) $p^A \in (v, 100]$ and $NOI \geq 0$; (ii) $p^A \in [0, v)$ and $NOI \leq 0$; and (iii) $p^A = v$ and any NOI. In any equilibrium, all agents achieve the same expected utility, as when $p^A = v$.*

Proof. See the Appendix.

3.2.2 Auction with Trading Frictions

We now turn to more realistic setups that include trading frictions. Our analysis in Section 3.1 shows that there can be a continuum of equilibria in the second stage, which makes solving for every equilibrium in a two-stage auction a daunting problem.

Instead of characterizing all of the equilibria, we show that in the presence of trading frictions, as outlined in Section 2.4, there exists a subset of equilibria of the two-stage game that results in bond mispricing. This result provides a resolution of the issue as to whether mispricing is possible in the two-stage CDS auction: it is.

Recall that without trading frictions, agents can undo any loss of utility that results from auction mispricing by optimally choosing between cash and physical settlement of their positions. If there are short-sale constraints (that is, Assumption 1 is imposed), agents with initial long CDS positions may not be able to do so because they are able to choose only b_i units of bonds for physical settlement. If, for at least one such agent $n_i > b_i$, agents with initial short CDS positions could become strictly better off, as a group, by pushing the price above v, because now they could purchase fewer bonds at a high price. Agents with long CDS positions and $n_i > b_i$, on the other hand, become worse off because they have to partially cash settle at a high price. This intuition is formalized in Proposition 3.

Proposition 3 *Suppose that only Assumption 1 is imposed and there exists an i such that*

$$n_i > b_i > 0, \tag{6}$$

Then, in the two-stage auction, $p^A \geq v$. In particular, there exists a subgame perfect overpricing equilibrium in which $p^A = 100$, and agents with initial short CDS positions achieve strictly greater utility than when $p^A = v$.

Proof. See the Appendix.

Proposition 3 shows that if only Assumption 1 is imposed, only overpricing equilibria can occur. Proposition 4 shows that if instead Assumption 2 holds, underpricing equilibria can also be realized. The amount of underpricing is related positively to the NOI.

Proposition 4 *Suppose that (i) Assumption 2 holds, (ii) there exist at least one protection seller who cannot hold defaulted bonds:*

$$\sum_{i:n_i>0} n_i + \sum_{i \in \mathcal{N}_+ : n_i<0} n_i > 0, \tag{7}$$

and (iii) for any $n_i > 0$

$$n_i > \frac{\sum_{j:n_j>0} n_j + \sum_{j\in\mathcal{N}_+:n_j<0} n_j}{K+1}, \tag{8}$$

where K is a total number of agents with initial long CDS positions. Then there exist a multitude of subgame perfect underpricing equilibria for the two-stage auction, in which

$$\text{(i)} \quad NOI > 0, \qquad \text{(ii)} \quad \frac{\partial p^A(NOI)}{\partial NOI} < 0, \tag{9}$$
$$\text{and} \qquad \text{(iii)} \quad 0 \le v - p^A(NOI) \le NOI \times \left|\frac{\partial p^A(NOI)}{\partial NOI}\right|.$$

In particular, there exists a subset of full-game equilibria where the second stage leads to a final price p^A that is a linear function of the NOI:

$$p^A = v - \delta \times NOI \ge 0 \tag{10}$$

for any NOI that can be realized in the first stage.

Proof. See the Appendix.

We give a formal proof by construction in the Appendix. In the proof, we show that if optimal physical settlement requests do not satisfy condition (5), there exist second-stage equilibria with $p^A \le v$, where agents play either linear strategies:

$$x_i(p) = \max\{a + b(v-p) - n_i + y_i, 0\}, \tag{11}$$

or non-linear strategies:

$$x_i(p) = \max\{c(v-p)^\lambda - n_i + y_i, 0\}, \tag{12}$$

a, b, c, and λ are specified in the Appendix. A similar set of strategies is used in Back and Zender (1993) to construct equilibria in a standard auction without CDS positions. There could also be other classes of equilibrium second-stage strategies. We use strategies (11) and (12) mainly because they lead to a closed-form solution. The main challenge in the rest of the proof is to solve jointly for equilibrium physical

settlement requests and the second-stage equilibrium price.

A closer inspection of (3) reveals that if the final auction price is lower than v and is not affected by agents' physical settlement requests (i.e., participants always choose to play same-price equilibria as long as the NOI is high enough to ensure second-stage underpricing), agents with long (short) CDS positions only have an incentive to choose full cash (physical) settlement in the first stage. This first-stage play entails that the NOI must be negative. As a result, second-stage underpricing equilibria in which $\partial p^A/\partial NOI = 0$ cannot be equilibria of the full game. However, if the strategies played in the second stage are such that the final auction price is a negative function of the NOI, the incentives of agents with long CDS positions become nontrivial. Submission by such agents of a physical settlement request could lead to a larger NOI and in turn to a lower final auction price, which would increase the payoff they receive from their partial cash settlement. The larger the initial positions of agents with long CDS positions, the stronger the incentives to lower the price via partial physical settlement. Condition (8) guarantees that the long positions of agents are sufficiently large to ensure that they choose physical settlement of enough CDS positions to render the resulting NOI positive.

The subset of equilibria characterized in Proposition 4 is the simplest and serves as an example of underpricing. There may be other equilibria that result in underpricing that we have not found. While condition (7) is necessary for an underpricing equilibrium to exist, condition (8) can be relaxed at the expense of a more complicated proof.

Propositions 3 and 4 show that there can be either underpricing or overpricing equilibria in the two-stage game with $NOI > 0$, if there are trading frictions. A similar set of results can be obtained for $NOI < 0$.

3.3 Auction with a Cap

We now discuss the implications of the imposition of a price cap, $IMM + s$, in the second stage. In the presence of the cap, mispricing in the auction depends on the bidding behaviour of dealers in the first stage. When there are no frictions, Proposition 2 shows that in all possible equilibria, all participants achieve the same utility as they would if $p^A = v$. Thus, dealers do not have any incentives to set their

optimal quotes, π_i, and therefore the IMM, to values other than v.

In the presence of frictions, there can be either underpricing or overpricing equilibria in the auction without a cap (Propositions 3 and 4). The cap cannot eliminate underpricing equilibria.[4] In addition, if the cap is set too low, that is, when $IMM + s < v$, equilibria with $p^A = v$ are ruled out.

The cap can, however, eliminate overpricing equilibria. As an illustration, consider a simple case in which all dealers have zero CDS positions. Proposition 3 shows that when there are short-sale constraints, the final auction price can be as high as 100 in the absence of a cap. Following the same logic as in Proposition 3, one can show that if the cap is greater than v, there exists an equilibrium with the final price equal to the cap. Given that in any such equilibrium dealers do not realize any profit, they may set their optimal quotes to v. Then, $IMM = v$ and $p^A = v + s$.

4 Empirical Evidence

We now consider whether empirical auction outcomes are consistent with the developed theory. We stress at the outset that our theoretical analysis does not account for all possible reasons for biases in the auction. For example, the presence of private asymmetric information concerning valuation, capital constraints, or risk-aversion of agents may affect the auction outcomes. We leave full theoretical and empirical analysis of the relative contribution and potential interaction between the different sources of inefficiencies as a project for further work. In section 5, we comment briefly on how the theory might be extended and conclude that the strategic bidding channel is robust to the inclusion of additional effects.

First, we describe our data and document basic facts about CDS auctions and their relation to the underlying bond markets. Second, our theory predicts many possible outcomes in equilibrium that can occur during the second stage of an auction. We provide evidence regarding which strategies are played in practice and whether

[4] If $IMM + s > v$, underpricing equilibria of the uncapped auction in linear strategies, (11), remain underpricing equilibria of the capped auction if agents submit all their previously uncapped portion of demand at and above the cap. As an example, consider an extreme case in which all dealers have large positive CDS positions and the conditions of Proposition 4 hold. Following the logic of Proposition 4, one can show that there exists a subgame perfect equilibrium in which $p^A = 0$ and $IMM = v$.

the resulting outcomes are consistent with the theoretical equilibria. Finally, our theoretical analysis shows that the price cap may affect the final auction outcomes. We study the circumstances under which this cap has an effect.

4.1 Data

Our data come from two primary sources. The details of the auction settlement process are publicly available from the Creditfixings website (www.creditfixings.com). As of December 2011, there had been 117 CDS and Loan CDS auctions (80 unique names), settling contracts on both US and international legal entities. The full universe of CDS auctions contains 75 auctions in which the net open interest was to sell, 32 auctions where the net open interest was to buy, and 10 auctions with zero net open interest.

We merge these data with bond price data from the Trade Reporting and Compliance Engine (TRACE) database to characterize the relationship between auction outcomes and the underlying bond values. TRACE reports corporate bond trades for US companies only. Thus, our merged dataset contains 26 auctions. Most of the auctions took place in 2009 and were triggered by the Chapter 11 event. In only four of the 26 auctions (Six Flags, General Motors, Dynegy and AMR) was the net open interest to buy ($NOI < 0$). Table 2 summarizes the results of the auctions for these firms by reporting net notional values, IMM, NOI and the final auction price.

Net notional values with respect to any credit name comprise the sum of the net protection bought by net buyers (or, equivalently, net protection sold by net sellers). These values constitute an estimate of the maximum possible transfer of net funds between net sellers and net buyers of single-name protection that could be required when a credit event occurs. Net notional values are provided by the Depositary Trust and Clearing Corporation (DTCC) on a weekly basis. We report the DTCC number for the last full week preceding an auction whenever available and denote it by $NETCDS$.

Table 3 provides summary statistics of the deliverable bonds for each auction for which we have bond data.[5] Deliverable bonds are specified in the auction protocols,

[5]A clarification regarding the auctions of Abitibi and Bowater is in order. AbitibiBowater is a corporation, formed by Abitibi and Bowater for the sole purpose of effecting their combination. Upon completion of the combination, Abitibi and Bowater became subsidiaries of AbitibiBowater

available from the Creditfixings website. To gain a perspective on how large NOI is we report the ratio of net open interest to the notional amount of deliverable bonds, obtained from the Mergent Fixed Income Securities Database and denoted by NAB. There is strong heterogeneity in NOI/NAB across different auctions, with absolute values ranging from 0.38% to 56.82%. In practice, NOI has never exceeded NAB.

Figure 2 shows the number of trades and their overall volume in the bond market in the days surrounding the auction. We scale daily volume by NAB and average across all auctions. Volume is about 0.5% of NAB before the auction. It increases six-fold on the day of the auction, stays relatively high for the next three days, and finally settles at about 1% of NAB. The total trading volume in the event window of -1 to +4 days is 10% of NAB. As a comparison, the average NOI is 13% of NAB. Therefore, the amounts of bonds traded in the bond market and in the auction are of comparable magnitudes. For each auction, we scale the number of trades on day t by the number of trades on the auction day (day 0) and average across all auctions. We can see that the number of trades follows the same patterns as daily volume, with most trades occurring around the auction. The average number of trades on the auction day across all auctions is 51.

We construct daily bond prices by weighing the price for each trade against the trade size reported in TRACE, as in Bessembinder, Kahle, Maxwell, and Xuet (2009). These authors recommend eliminating all trades under $100,000, because these are likely to be noninstitutional. The larger trades have lower execution costs; hence, they should reflect the underlying bond value with greater precision.

For each company, we build a time-series of bond prices. We select an event window of -8 to +12 days. The left boundary is determined by the shortest time between a credit event and an auction in our sample. The choice of the right boundary is dictated by bond liquidity, which generally declines after the auction. We scale the representative bond prices by the final auction price to compare the two. Figure 3(a) displays daily bond prices normalized by the auction final price, p_t/p^A, equally weighted across the auctions for which we have reliable bond data.[6]

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and the businesses that were formerly conducted by Abitibi and Bowater became the single business of AbitibiBowater. The CDS contracts were linked to the entities separately, and, as a result, there were two separate auctions.

[6]We exclude the auction for Charter, which has only 10 trades in the [-8,0] window. Of these 10 trades, only 6 are of magnitude greater than $1MM. As a reference, the second-worst company

We display the cases with the $NOI > 0$ and the $NOI < 0$ separately in the top and bottom panels, respectively. We see that when the $NOI > 0$, the price generally declines, reaches its minimum on the auction day, then reverts to its initial level. The pattern is reversed when the $NOI < 0$. Finally, to complement these figures with auction-specific information, the last column of Table 3 reports a weighted average bond price on the day before the auction, p_{-1}.

4.2 Bidding Behaviour at the Second Stage

Our analysis in Section 3.2.2 shows that auction participants may use many possible bidding strategies resulting in multiple equilibrium outcomes. In consequence, we start by establishing how participants bid in practice. We use auctions bids provided by Creditfixings.

Recall that auction participants submit their bids through dealers. Two issues must be noted. First, while all bids are reported, only the dealer who facilitated the particular bid submission is revealed. Second, participants may submit their bids through multiple dealers. As a result of these aspects of the structure of bidding, we do not have full information about the bidding strategies of each individual participant. In what follows, we will be treating all the bids submitted by each dealer as an individual bidding schedule, so our results have to be interpreted with the above caveats in mind.

Figure 4 illustrates typical bids using the example of Washington Mutual. Each line represents demand schedule $x_i(p)$, normalized by the NOI, submitted by a participating dealer. Each schedule shows how many bonds a participant i is willing to buy at each price p. The Figure shows that each individual bid schedule resembles a discretized version of a linear schedule. This prompts us to evaluate this functional form in a systematic way.

We follow Hortaçsu (2002) in our analysis. We estimate the following specification for each bidder's price-quantity pairs:

$$p_{ijk} = \alpha_{ij} + \beta_{ij} x_{ijk} + \varepsilon_{ijk}, \tag{13}$$

in terms of data reliability, Chemtura, has 35 trades, the amounts of all of which are greater than \$1MM.

where $\{p_{ijk}, x_{ijk}\}$, is the k-th price-quantity pair submitted by dealer i in auction j. An average dealer across the 26 auctions submits 7 price-quantity pairs. To ensure meaningful results, we restrict our analysis to dealers who submit at least three price-quantity pairs, $\max\{k_{ij}\} \geq 3$, as in Hortaçsu (2002). The overall fit, as measured by the average (median) of R^2's over all individual bid functions, is 0.85 (0.90) for this specification. These numbers are very similar when we separate the analysis by the sign of the NOI, or whether the final price is capped or not. These results suggest that agents use approximately linear bidding strategies in the second stage of the auction.[7]

4.3 Auction Outcomes

4.3.1 Establishing a Proxy for the Fair Value

In order to study the auction outcomes, we have to be able to measure the extent of mispricing. The fair value v is never observed. Because of this, we use available bond prices from the bond market to construct a proxy. Our approach is similar to that of studies that examine mispricing during IPOs, SEOs, or Treasury auctions. In the case of IPOs, researchers use the end-of-the-day stock price as a benchmark for establishing what the price should have been at IPO (Ibbotson, 1975). For SEOs, researchers use close-to-offer, or offer-to-close changes, that is, compare the new prices to the parallel market in the corresponding security (Smith, 1977). In the latter case, a when-issued price or a price of an existing bond with characteristics that are similar to the ones of the bond being auctioned is used (Goldreich, 2007 and Lou, Yan, and Zhang, 2012).

Figure 3(a) shows that the representative bond price on the day of the auction, p_0, is the most conservative benchmark against which to assess the magnitude of potential underpricing during the auction. One may be concerned that the construction of p_0 involves bond prices after the auction, so the best conservative measure that avoids the look-ahead bias would be the bond price from the day before the auction, p_{-1}. These two numbers are our main reference points, but we will check the robustness

[7] In the universe of all 117 auctions, an average dealer submits 4.07 price-quantity pairs. The average (median) of R^2's over all individual bid functions, is 0.86 (0.91). The number of price-quantity pairs is smaller for the whole universe because many European auctions had small NOI.

of our findings with respect to these choices and will comment on which one is more appropriate, depending on specific elements of our analysis.

We can think of three potential concerns about the appropriateness of bond prices as a benchmark. First, one may argue that one should not use bond prices as a proxy for v, because if they were a good proxy, there would be no need to conduct an auction (as a price discovery mechanism): the CDS could be settled in cash using the average transaction prices. However, if using the average transaction prices were the settlement rule, this would create strong incentives for market participants to manipulate the corresponding bond prices. As a result, the average transaction prices would not reflect the desired fundamental value. Therefore, it is precisely because the CDS are not settled on the basis of market bond prices that these prices can be informative about the fundamental value.

Second, it is conceivable that the auction process establishes a v that differs from our proposed benchmarks simply as a result of the efficient centralized clearing mechanism of the auction. However, the V-shaped pattern of deviation between the bond and auction prices that is shown in Figure 3(a) alleviates this concern. If v were close to p^A, one would expect average bond prices to remain in the region of the auction price after the auction, whereas in practice they return to the pre-auction levels.

Third, agents will likely use only cheapest-to-deliver bonds for physical delivery. As a result, our method for approximating the fair value might overestimate it. This argument is not applicable when the credit event is Chapter 11, and all the deliverable bonds are issued by the holding company and cross-guaranteed by all subsidiaries. In Chapter 11, bonds with no legal subordination are treated as identical; see, for example, Guha (2002).[8] The reasons for this are that all the bonds stop paying coupons and mature (cease to exist) at the same time, with identical terminal payouts to all bondholders. Hence, there need be no concern that some bonds are cheaper to deliver due to the difference in their fundamental value.

As an example, Figure 5 shows weighted daily prices of each individual Washington Mutual bond issue, identified by its CUSIP. We see that there are large differences between the prices of different bonds in the period leading to the credit event (trading day -19). After this day, the prices of all bonds are very similar. The prices cannot be

[8]CDS contracts on bonds with different seniorities are settled in different auctions. Examples of this in our data are the Dura/Dura Sub auctions.

identical because trades may occur at different times of the day, and because trades may be either buyer- or seller-initiated, which means prices will be closer to bid or ask prices, respectively.

In our sample, 16 out of 26 credit events are triggered by Chapter 11 bankruptcy and have one issuer. These companies should not have bonds that diverge in value. Nonetheless, we confirm empirically that this is indeed the case. There are three companies that filed for Chapter 11 and have multiple subsidiaries issuing bonds, but for which TRACE contains trade data for only one subsidiary in the event window (CIT, Lyondell, and Quebecor). We treat these three names the same way as the 16 firms without subsidiaries.

There are four companies that filed for Chapter 11 and have multiple subsidiaries, and where we have data for the bonds of these subsidiaries (Bowater, Charter, Nortel and Smurfit-Stone). In all of these cases, the bonds of the different subsidiaries are legally pari-passu with each other. However, some of them may be structurally subordinated to others, so they could be cheaper. For this reason, we select the cheapest bonds in the case of these four companies (but the differences are not large in practice). There are three companies with a credit event other than Chapter 11 (Abitibi, Capmark and Rouse) in which we also select the cheapest bonds.

Finally, to account for other potential issues regarding the selection of deliverables that could work against our findings, we treat the aforementioned differences in bond prices (which are due to bid-ask spread and timing differences) as real differences, and select the lowest-priced bonds. Specifically, we take representative daily prices of a company's deliverable bonds to be equal to the weighted daily prices of their bond issues with the lowest average pre-auction price over the [-5,-1] window, provided that trading in these bond issues is fairly active.[9] The results are presented in Figure 3(a). It can be seen that even with these conservative criteria for selecting bonds, a similar V pattern remains.[10]

[9]The requirement is that the trading volume over the 5 trading days before the auction constitutes at least 5% of total trading volume for the company.

[10]Gupta and Sundaram (2011) address the cheapest-to-deliver issue using an alternative procedure that is based on econometric modelling of issue-specific pricing biases, and arrive at similar conclusions.

4.3.2 Price Impact at the Second Stage

Figure 3(a) shows that there is underpricing, on average, during auctions when the $NOI > 0$ and mild overpricing, on average, when the $NOI < 0$, regardless of the bond price that we use as a reference. This evidence is consistent with Proposition 4. In addition, part (ii) of the Proposition establishes that the final auction price should be a negative function of the NOI for the underpricing equlibria to realize. Thus, as a next step, we evaluate whether this relationship is observed in the data.

Proposition 4 characterizes the relationship between p^A and the NOI within each auction (for a given name). However, in the data, we do not have repeated realizations of the (p^A, NOI) pair for the same auction. Instead, we observe the (p^A, NOI) pairs across auctions. In general, different equilibria can be realized across auctions. This means that sensitivity of the auction price to the NOI could vary across auctions, and, in particular, can be related to various auction characteristics. Therefore, in our empirical analysis, we scale the (p^A, NOI) pairs to make them more homogenous, as if they were observations from the same auction. In particular, we scale p^A by the respective bond price to express prices in per cent rather than in dollars. We use p_{-1}, but subsequently check the robustness of our conclusions to this choice. We do not have firm theoretical guidance about scaling the NOI, so we try several options: (i) no scaling; (ii) our estimate N of the number of participants from the set $\mathcal{N}$; (iii) $NETCDS$; and (iv) NAB.[11]

Figure 6 displays p^A/p_{-1} against the scaled NOI. We see that, regardless of the normalization, the sign of the relationship is negative, as is predicted by theory. When NOI is not scaled by anything, we note two extreme cases: Lehman and Tribune. Tribune is extreme because of the magnitude of underpricing. Lehman has the largest NOI by far. Various normalizations of the NOI produce qualitatively similar results, with Tribune standing out across all versions. These results suggest the following.

First, they suggest that to produce a more conservative estimate of average underpricing one should exclude the Tribune auction. Figure 3(b) summarizes this case. We conclude that the most conservative estimate of average underpricing is 6%. This is an economically significant number. It is much larger than the bond liquidity pre-

[11] We use the fact that dealers submit their own and customers' price-quantity pairs separately. Therefore, our estimate N is constructed as the sum across dealers of the maximum number of price-quantity pairs that are submitted at the same price.

mium. Dick-Nielsen, Feldhütter, and Lando (2012) document that *annual* liquidity premium on speculative grade bonds averaged 58 basis points and peaked at 197 basis points during the Lehman default. Also, as a comparison, we note that US Treasury bond auctions result in underpricing of one or two basis points (Goldreich, 2007). SEO underpricing averages 2.2% (Corwin, 2003). The average IPO underpricing has varied over the years, but, excluding the dot-com period, IPO underpricing was 7% during 1980-1989, doubling to almost 15% during 1990-1998 and then reverting to 12% during the period of 2001-2003 (Loughran and Ritter, 2004). The IPO numbers are higher than others, but of the same order of magnitude.

Second, all the panels of Figure 6 are suggestive of an approximately linear relationship between the underpricing and the NOI, as in equation (10) of Proposition 4. We quantify this relationship using a univariate cross-sectional regression of p_{-1}/p^A on NOI/S, where S represents one of the four scalings (1, N, $NETCDS$, or NAB)

$$p^A/p_{-1} = \alpha + \beta \times NOI/S + \varepsilon. \tag{14}$$

Table 4 reports the results of OLS regressions. In all cases β is significantly negative, α is close to 1, and R^2 ranges between 10% (when there is no scaling) to 49% (when the NOI is scaled by NAB)[12]. Table 4 also reports the results of median regressions to make sure that the results are not driven by extreme observations. In all cases, the results are very close to those from the OLS regressions.

These results are consistent with Proposition 4, which shows that there exist second-stage equilibria in which the final price, p^A, depends linearly on the NOI (equation (10)). Given that the only theoretical restriction on the slope, δ, is its sign, the linear relationship (10) can be written as

$$p^A/v = 1 + [\beta/S] \times NOI, \quad \beta < 0.$$

If agents use equilibrium strategies with the same β across auctions, the estimated cross-sectional regression β will also be an estimate of the intra-auction relationship. While the assumption that the same linear dependence obtains across auctions is admittedly strong, it can be accommodated by the following considerations. If all

[12]The slope β is significant at the 10% level when $S = 1$. If we remove the Lehman datapoint, $\beta = -0.26$ and is significant at 5% level, $R^2 = 16.48\%$.

agents in an auction take historical information about previous types of equilibria into account when forming their beliefs, β is unlikely to vary much across auctions. In addition, the core auction participants (the dealers) remain the same and are likely to use strategies that generate similar equilibria across auctions. Finally, the estimated α is insignificantly different from 1, which is again consistent with the theory.

As a robustness check, we replace p_{-1} in (14) by any other price from the event window. Because liquidity is low outside the immediate vicinity of the auction, we construct a representative price by binning trading days. Bin -2 combines days from -8 to -4; bin -1 combines days from -3 to -1; bin 0 is day 0; bin 1 combines days from 1 to 3; bin 2 combines days from 4 to 8. Figure 7 shows the corresponding time series of α and β for the case of $S = NAB$. The results are similar to regression (14), although the relationship between p^A and the NOI weakens towards the end of the event window. The conclusions for other choices of S are qualitatively similar.

4.4 The Impact of the Cap

Our results in Section 3 show that when the final price, p^A, is capped it can be either above or below the true value of the bond, v. In the former case, the cap is beneficial because it prevents strong overpricing. In the latter case, the cap is detrimental because the auction price cannot reach the fair value.

Our analysis suggests a way of differentiating between the two cases. Consider outcomes in which $NOI > 0$ (the case that considers outcomes in which $NOI < 0$ follows similar reasoning). According to Proposition 1, the price can be higher than v if, after the first stage, the aggregate short net CDS position of agents participating in the second stage is larger than the net open interest. In this case, protection sellers have an incentive to bid above the true value of the bond to minimize the amount paid to their CDS counterparties. Notice that while bidding at a price above v, they would like to minimize the amount of bonds acquired at the auction for a given final auction price. Thus, if the price is above v, they will never bid to buy more than NOI units of bonds.

The case in which p^A is capped and lies below the true value of the bond is brought about when dealers set IMM so that $IMM + s$ is below v. This prevents the agents

from playing second-stage equilibrium strategies with the final price above the cap. In this case, submitting a large demand at the cap price leads to greater profit. Thus, in the presence of competition and sharing rules, agents have an incentive to buy as many bonds as possible and would bid for substantially more than NOI units.

The final price is capped in 25 of the 117 credit-event auctions.[13] Figure 8 shows the entities and the individual bids at the cap price. The individual bids are represented by different colours, and bid sizes are scaled by NOI to streamline their interpretation. For example, there are seven bids at the cap price in the case of General Growth Properties. Six of these are equal to NOI and the seventh one is approximately one-fourth of NOI.

We can see that in all but two auctions (Kaupthing Bank and Glitnir), the bids at the price cap do not exceed NOI. The results suggest that in these cases, the final auction price is likely higher than the true bond value. Of the 25 auctions with a capped price, we have bond data for only five companies: Smurfit-Stone, Rouse, Charter Communications, Capmark and Bowater. Comparing the final auction price from Table 2 with the bond price from Table 3, we can see that that the bond price (our proxy for the true bond value) is lower than the final auction price for these five companies, as expected.

5 Extensions

Section 4 documents our finding that when NOI is large, the auction generally results in a price considerably below the fair value. We now suggest several modifications to the auction design that can reduce mispricing, and discuss some of the assumptions of the model.

5.1 Allocation Rule at the Second Stage

As usual, we focus on the case of $NOI > 0$. Proposition 1 shows that if condition (5) does not hold, the CDS auction is similar to a 'standard' auction, so the price can be below v. Kremer and Nyborg (2004b) show that in a setting without CDS positions,

[13] Of these 25 auctions, four (Ecuador, Anglo Irish Bank (Restructuring), Allied Irish Bank, and Anglo Irish bank) have a negative NOI. So the above discussion for the case of positive NOI should be adjusted appropriately for them.

a simple change of the allocation rule from pro-rata on the margin (2) to 'pro-rata' destroys all underpricing equilibria, so that only $p^A = v$ remains. Under the pro-rata rule, the equilibrium allocations q_i are given by

$$q_i(p^A) = \frac{x_i(p^A)}{X(p^A)} \times NOI.$$

That is, the total rather than marginal demand at p^A is rationed among agents. The next proposition extends the result of Kremer and Nyborg (2004b) to our setting. We demonstrate that if $IMM + s \geq v$, the second-stage equilibrium price p^A cannot be less than v. This is true even if the agents are allowed to hold nonzero quantities of CDS contracts.

Proposition 5 *Suppose that the auction sharing rule is pro-rata. In this case, if $NOI > 0$ then $p^A \geq \min\{IMM + s, v\}$. If $NOI < 0$ then $p^A \leq \max\{IMM - s, v\}$.*

Proof. See Appendix.

To develop intuition for this result, consider the case of positive NOI. According to Proposition 1, if condition (5) does not hold, the allocation rule of pro-rata on the margin may inhibit competition and lead to underpricing equilibria. The presence of agents who hold short CDS contracts does not help in this case. The pro-rata allocation rule (i) does not guarantee the agents their inframarginal demand above the clearing price, and (ii) ties the proportion of allocated bonds closely to the ratio of individual to total demand at the clearing price. Therefore, a switch to such a rule would increase competition for bonds among agents. If $p^A < v$, demanding the NOI at a price only slightly higher than p^A allows an agent to capture at least half of the surplus.[14] As a result, only fair-price equilibria survive.

If agents have capacity constraints and cannot absorb large quantities of bonds, deviations that destroy underpricing outcomes in the proof of Proposition 5 may become infeasible. In these cases, we can instead use a hybrid allocation rule (see Kremer and Nyborg (2004a)), which is the average of the pro-rata and pro-rata on

[14] At $p^* > p^A$, the demand of other agents $X_{-i}(p^*) < NOI$ which guarantees at least half of the NOI to an agent who submits $x_i(p^*) = NOI$.

the margin rule: when $X(p^A) > NOI$,

$$q_i(p^A) = \frac{1}{2}\frac{x_i(p^A)}{X(p^A)} \times NOI + \frac{1}{2}\left(x_i^+(p^A) + \frac{x_i(p^A) - x_i^+(p^A)}{X(p^A) - X^+(p^A)} \times \left(NOI - X^+(p^A)\right)\right).$$

In the presence of CDS positions, the hybrid allocation rule results in the only equilibrium final price $p^A = v$.[15]

5.2 The Price Cap

Again, we consider the case of $NOI > 0$. Our theoretical analysis in Section 4.4 shows that the presence of a price cap can result in auction outcomes with mispricing that is either lower or higher than the fair value. The cap is likely to help when NOI is small and the temptation to manipulate the auction results to obtain overpricing is greatest. At the same time, the cap allows dealers to limit the final price to below v at the second stage. In this case, setting a larger cap can ensure that v is still feasible as an outcome even if IMM turns out to be low.

These results suggest that making the cap conditional on the outcome of the first stage of a CDS auction can lead to better outcomes. In our base model without uncertainty, the optimal conditional cap is trivial. If $IMM < v$, setting $s^* = v - IMM$ ensures that the set of second-stage equilibria includes v. If $IMM \geq v$, it is best to set $s^* = 0$. While the conditional cap cannot eliminate the worst underpricing equilibria, it can ensure that agents who want to bid aggressively will be able to do so.

In practice, v is unobservable. A sensible alternative, then, is to make the cap conditional on IMM and a proxy for v, e.g., set $s^* = \max\{0, p_{-1} - IMM\}$. In this case, however, one has to be concerned that the use of bond market prices to cap cash settlement prices may induce bond price manipulation. Therefore, one may consider using NOI as a conditioning variable instead. For example, set $s^* = \phi(NOI/S)$, where S is one of the scalings considered in Section 4 and $\phi > 0$ is an increasing (e.g., linear) function chosen by the auctioneer. Then, if NOI/S is small, the likely overpricing can be mitigated by a smaller cap. If NOI/S is large, the larger cap should be large enough to ensure that v is a feasible outcome. We leave explicit

[15] The proof is similar to that of Theorem 3 in Kremer and Nyborg (2004a) and is available upon request.

modeling of different conditional caps and their impact on CDS auction outcomes for future research.

5.3 Risk-averse agents

So far, we have considered only risk-neutral agents, so that we could conduct our analyses in isolation from risk. If agents are risk-averse, the reference entity's risk is generally priced. Even though a CDS is in zero net supply, its settlement leads to a reallocation of risk among the participants in the auction; hence, it can lead to a different equilibrium bond price. In particular, when NOI is large and positive, and there are only a few risk-averse agents willing to hold defaulted bonds, the auction results in highly-concentrated ownership of the company's risk and can thus lead to a lower equilibrium bond price.

Notice, however, that risk-aversion does not automatically imply a lower auction price. For example, if marginal buyers of bonds in the auction are agents who previously had large negative CDS positions (as in Proposition 4), their exposure to risk after the auction may actually decrease. As a result, they could require a lower risk premium.

We do not have data on individual agents' bids and positions, so we cannot determine whether the observed deviation in the auction price from the OTC bond prices is due to mispricing equilibria or risk-aversion. It is likely that both factors work together in the same direction. Data on individual agents' bids and positions could help to quantify the effect of the two factors on the observed relationship between the auction price and the size of net open interest.

5.4 Private information

So far, we have restricted our attention to the simplest case in which agents' CDS positions are common knowledge. This may seem to be a very strong assumption, given that CDS contracts are traded in the OTC market. Notice, however, that in the type of equilibria constructed in Propositions 3 and 4 (linear case) and, conditions (6) and (7), (8) completely define the two equilibria. Therefore, Propositions 3 and 4 continue to hold with private CDS positions as long as conditions (6) and (7), (8)

are public knowledge.[16] One can argue that this is likely to be the case. For example, (6) assumes that there exists an agent whose long position in CDSs is larger than her bond holdings. Given the much larger size of CDS contracts compared to the value of bonds outstanding, (6) holds as long as aggregate long CDS positions are larger than the value of the outstanding bonds.

We also assume that agents value bonds identically, and that this value is common knowledge. This assumption provides a stark benchmark: we are able to show that the auction results in mispricing even in such a basic case. We conjecture that the current auction mechanism would be even less able to arrive at the fair value when agents have private or heterogeneous valuations.

6 Conclusion

We have presented a theoretical and empirical analysis of the settlement of CDS contracts when a credit event takes place. A two-stage, auction-based procedure aims to establish a reference bond price for cash settlement and to provide market participants with the option to replicate an outcome for physical settlement. The first stage determines the net open interest (NOI) in the physical settlement and the auction price cap (minimum or maximum price, depending on whether the NOI is to sell or to buy). The second stage is a uniform divisible good auction with a marginal pro-rata allocation rule that establishes the final price by clearing the NOI.

In our theoretical analysis, we show that the auction may result in either overpricing or underpricing of the underlying bonds. Our empirical analysis establishes that underpricing is more common in practice. Bonds are underpriced on average, and the amount of underpricing increases with the NOI. We propose introducing a pro-rata allocation rule and a conditional price cap to mitigate this mispricing.

[16] The formal proofs follow closely the original proofs for the full information case and are available upon request.

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Appendix

Proof of Proposition 1

Suppose that $p^A < v$ and condition (5) holds. We show that this cannot be true in equilibrium. Let the equilibrium allocation of bonds to agent i be q_i. Consider a change in the demand schedule of player i from x_i to x_i' that leads to the auction price $p \in [p^A, v]$. Let q_i' denote the new bond allocation to agent i. Since demand schedules are non-decreasing, $q_i' \geq q_i$. Agent i's change in profit is thus

$$\Delta_i = \left[(v - p^A)q_i - (n_i - y_i)p^A\right] - \left[(v - p)q_i' - (n_i - y_i)p\right] =$$

$$= (p - p^A)(n_i - y_i + q_i) - (v - p)(q_i' - q_i) \leq (p - p^A)(n_i - y_i + q_i). \tag{A1}$$

The equilibrium conditions require that $\Delta_i \geq 0$ for all i. Summing over all i such that $n_i < 0$, it must be that

$$0 \leq \sum_{i:n_i - y_i < 0} \Delta_i \leq (v - p^A) \sum_{i:n_i - y_i < 0} (n_i - y_i + q_i)\,.$$

Due to the fact that all $q_i \geq 0$,

$$\sum_{i:n_i - y_i < 0} (n_i - y_i + q_i) \leq \sum_{i:n_i - y_i < 0} n_i - y_i + NOI \leq 0, \tag{A2}$$

where we use (5). Thus, in any equilibrium with $p^A < v$, it must be that $\Delta_i = 0$ for all i with $n_i < 0$. (A1) and (A2) then imply that for any deviation x_i' that leads to $p \in [p^A, v]$, so it must be that $q_i' = q_i$. This is true for any $p \in [p^A, v]$, so the initial total demand $X(p)$ must be constant over $[p^A, v]$, and therefore $p^A = v$. Thus, we arrive at a contradiction.

Finally, suppose that condition (5) does not hold and there exists an equilibrium with $p^A > v$. Then there also exists an i such that agent i's equilibrium second-stage allocation $q_i > |n_i - y_i|$. Consider a variation of this agent's demand schedule, in which she submits zero demand at $p^A > v$ and demand equal to the NOI at $p^A = v$. Given this variation, the new auction price will be higher than or equal to v. Thus,

her profit increases by at least $(p^A - v)(q_i + n_i - y_i) > 0$, so $p^A > v$ cannot be an equilibrium outcome. QED.

Proof of Proposition 2

First, we show that when there are no trading frictions, in any equilibrium, (5) always holds. From the definition of the NOI,

$$\sum_{i:n_i<0}(n_i - y_i) + NOI = \sum_{i:n_i<0}(n_i - y_i) + \sum_i y_i = \sum_{i:n_i<0} n_i + \sum_{i:n_i>0} y_i \leq \sum_{i:n_i<0} n_i + \sum_{i:n_i>0} n_i = 0.$$

Proposition 1 then implies that if $NOI \geq 0$, $p^A \in (v, 100]$, and similarly, if $NOI \leq 0$, $p^A \in [0, v)$. Suppose that $p^A \in (v, 100]$. Clearly, only agents with positive remaining CDS positions after the first stage of the auction will be willing to buy bonds at a price above v. Agents with initial long CDS positions are allocated zero bonds.

We first consider agents' optimal physical requests and their utilities in a pure-strategy equilibrium. From (3), each of the utility functions of agents with initial long CDS positions will be

$$\Pi_i = n_i(100 - v) + (n_i - y_i)(v - p^A) + b_i v. \tag{A3}$$

If $p^A > v$, utility (A3) is maximized if $y_i = n_i$. Therefore, $\Pi_i = n_i(100 - v) + b_i v$ for $n_i > 0$. Thus, in any such equilibrium, agents with initial long CDS positions choose physical delivery, are allocated zero bonds, and achieve the same utility. The NOI is

$$NOI = \sum_i y_i = \sum_{i:n_i>0} n_i + \sum_{i:n_i<0} y_i = -\sum_{i:n_i<0}(n_i - y_i) \geq 0. \tag{A4}$$

In other words, the NOI is equal to the sum of outstanding CDS positions (after the first stage) held by agents with initial short CDS positions. From (3), the utility of agents with initial short CDS positions is given by

$$\Pi_i = n_i(100 - v) + (n_i - y_i + q_i)(v - p^A) + b_i v. \tag{A5}$$

Due to the fact that every agent can always guarantee utility $\Pi_i = n_i(100 - v) + b_i v$ by choosing physical delivery, q_i cannot be higher than $-(n_i - y_i)$. In addition, (A4)

implies that q_i cannot be lower than $-(n_i - y_i)$. Therefore, $q_i = -(n_i - y_i)$ and $\Pi_i = n_i(100 - v)$ for each $i : n_i < 0$.

In the case when $NOI \leq 0$ and $p^A \in [0, v)$, agents with initial short CDS positions choose physical delivery at the first stage and do not sell bonds at the second stage. The proof is similar.

Notice that the auction is a zero-sum game for participants (discarding utility terms which are not affected by auction outcomes). Each participant can guarantee himself a zero utility by choosing full physical settlement. Therefore, even in a mixed-strategy equilibrium, every participant attains the same utility. *QED*.

Proof of Proposition 3

The proof is by construction. We assume that there are at least two protection buyers with nonzero bond holdings. We construct an equilibrium in which $p^A \equiv 100$ and does not depend on individual cash and physical settlement choice. As in Proposition 2, if $p^A \equiv 100$, agents who hold initially long CDS contracts will choose physical delivery, and only agents with negative remaining CDS positions after the first stage will be willing to buy bonds in the auction. Proposition 1 shows that for any $NOI > 0$, if condition (5) holds (which turns out to be the case in the constructed equilibrium), $p^A = 100$ is an equilibrium of the second stage if agents use the following strategies:

$$x_i(p): \quad \begin{cases} x_i = NOI \times (n_i - y_i)/(\sum_{j:n_j<0}(n_j - y_j)) & \text{if} \quad v < p \leq 100, \\ x_i = NOI & \text{if} \quad p \leq v. \end{cases}$$

for agents with net negative CDS positions after they have submitted their physical settlement requests, and $x_i(p) \equiv 0$ for other agents. The profit earned by agent i with $n_i < 0$ is therefore

$$\Pi_i = \left(y_i - NOI \frac{n_i - y_i}{\sum_{j:n_j<0}(n_j - y_j)} \right)(100 - v) + b_i v. \tag{A6}$$

Taking the F.O.C. at $y_i = 0$, one can verify that it is optimal for agents with initial short CDS positions to choose cash settlement. Thus, $NOI = \sum_{j:n_j>0} \min\{n_i, \max\{b_j, 0\}\}$

and the profit accruing to any agent i with an initial short CDS position $n_i < 0$ is

$$\Pi_i = (100 - v)n_i \times \frac{-\sum_{j:n_j>0} \min\{n_i, \max\{b_j, 0\}\}}{\sum_{j:n_j<0} n_j} + b_i v > (100 - v)n_i + b_i v,$$

where the expression on the right hand side is the agent's utility if p^A is equal to v. *QED*.

Proof of Proposition 4

The proof is by construction. We construct a subgame perfect two-stage equilibrium in which the final auction price is a decreasing function of the NOI. In a manner similar to that reported in Kremer and Nyborg (2004b), it can be shown that one's attention can be restricted without loss of generality to equilibria in differentiable strategies. For simplicity, we provide the proof for the case in which agents have large long CDS positions. Specifically, we assume that for all $i : n_i > 0$:

$$n_i \geq NOI. \tag{A7}$$

Under this additional assumption, we can solve for the equilibrium in closed form. The general case follows similar reasoning, except that the number of the agents who submit nonzero demand for bonds at the second stage depends on the configuration of CDS positions. When A7 holds, only agents with non-positive CDS positions receive nonzero allocations in the equilibrium.

The proof consists of several steps. In step 1, we derive the F.O.C. for the optimal strategies at the second stage, given the remaining CDS positions of the agents after the first stage. In step 2, we derive the F.O.C. for the optimal physical settlement requests.[17] In step 3, we show that the second-stage equilibrium with price p^A can be supported if agents use the following second-stage strategies:

$$x_i(p) = \max\{a + b(v - p) - n_i + y_i, 0\},$$

[17] In principle, agents can potentially make large deviations in physical settlements that can change the sign of the NOI. They will not do so if the out-of-equilibrium final price, $p^A \geq v$ for $NOI < 0$.

$$x_i(p) = \max\{c(v-p)^\lambda - n_i + y_i, 0\}$$

(a, b, c, and λ are specified later). In step 4, we solve for agents' optimal requests for physical settlement, given the above second-stage strategies. Finally, we solve for the NOI.

Step 1. Recall that at the second stage, player i solves problem (4):

$$\max_{x_i(p)} \; (v - p(x_i(p), x_{-i}(p)))\, q_i(x_i(p), x_{-i}(p)) + (n_i - y_i) \times (100 - p(x_i(p), x_{-i}(p)))\,.$$

In any equilibrium of the second stage, the sum of the demand of agent i, $x_i(p^A)$, and the residual demand of the other players, $x_{-i}(p^A)$, must equal the NOI. Therefore, solving for the optimal $x_i(p)$ is equivalent to solving for the optimal price, p^A, given the residual demand of the other players. Thus, the F.O.C. for agent i at the equilibrium price, p^A, can be written as

$$(v - p^A)\frac{\partial x_{-i}(p^A)}{\partial p} + x_i(p^A) + n_i - y_i = 0 \qquad \text{if} \qquad x_i(p^A) > 0, \tag{A8}$$

$$(v - p^A)\frac{\partial x_{-i}(p^A)}{\partial p} + x_i(p^A) + n_i - y_i \geq 0 \qquad \text{if} \qquad x_i(p^A) = 0. \tag{A9}$$

Step 2. Recall that agent i's profit is given by equation (3):

$$\begin{aligned}\Pi_i \;=\; & \underset{\text{auction-allocated bonds}}{(v - p^A)q_i} + \underset{\text{remaining CDS}}{(n_i - y_i) \times (100 - p^A)} \\ & + \underset{\text{physical settlement}}{100 y_i} + \underset{\text{remaining bonds}}{v(b_i - y_i)}\,.\end{aligned}$$

Using the fact that $\partial NOI/\partial y_i = 1$, we have that the F.O.C. for the optimal settlement amount, y_i, for agent i, satisfies

$$\frac{\partial \Pi_i}{\partial y_i} = 0 \quad \text{if} \quad y_i \neq 0 \quad \text{and} \quad y_i \neq n_i, \tag{A10}$$

$$\frac{\partial \Pi_i}{\partial y_i} \leq 0 \quad \text{if} \quad y_i = 0 \quad \text{and} \quad n_i > 0, \quad \text{or} \quad y_i = n_i \quad \text{if} \quad n_i < 0, \tag{A11}$$

$$\frac{\partial \Pi_i}{\partial y_i} \geq 0 \quad \text{if} \quad y_i = 0 \quad \text{and} \quad n_i < 0, \quad \text{or} \quad y_i = n_i \quad \text{if} \quad n_i > 0, \tag{A12}$$

where

$$\frac{\partial \Pi_i}{\partial y_i} = -\frac{\partial p^A(NOI)}{\partial NOI}(n_i - y_i + q_i) - (v - p^A(NOI))\left(1 - \frac{\partial q_i}{\partial y_i}\right). \tag{A13}$$

Step 3. Let M be the number of agents with nonpositive CDS positions who are allowed to hold bonds, and let $\lambda = 1/(M-1)$. Then consider the following set of strategies at the second stage:

$$x_i(p) = \max\left\{\frac{NOI + \sum_{j\in\mathcal{N}_+:n_j<0}(n_j - y_j)}{M(M-1)}\left(M - 2 + \frac{v-p}{v - p^A(NOI)}\right) - n_i + y_i, 0\right\}. \tag{A14}$$

and

$$x_i(p) = \max\left\{\frac{NOI + \sum_{j\in\mathcal{N}_+:n_j<0}(n_j - y_j)}{M}\frac{(v-p)^\lambda}{(v - p^A(NOI))^\lambda} - n_i + y_i, 0\right\}. \tag{A15}$$

Demand schedules (A14) and (A15) imply that agents with nonpositive CDS positions who are allowed to hold bonds receive, at $p = p^A$, the following bond allocations:

$$q_i = \frac{NOI + \sum_{j\in\mathcal{N}_+:n_j<0}(n_j - y_j)}{M} - (n_i - y_i). \tag{A16}$$

Equation (A9) implies that agents with initial long CDS positions receive zero equilibrium bond allocations at the second stage, as long as

$$n_i - y_i \geq \frac{NOI + \sum_{j\in\mathcal{N}_+:n_j<0}(n_j - y_j)}{M-1}. \tag{A17}$$

If this is the case, equation (A8) implies that strategies (A14) and (A15) form an equilibrium at the second stage, with the equilibrium price equal to p^A.[18]

[18] Technically, one extra condition is needed to ensure the existence of the constructed equilibrium. Inequality (A17) must continue to hold for every possible deviation $\hat{y}_j > y_j$ by each participant $j \in \mathcal{N}$. If, for some such $\hat{y}_j$, this condition breaks down for agent i with a long CDS position, this agent will participate in the second stage of the auction, which could increase the profit earned by agent j. (Of course, agent i could increase y_i itself, in which case $j = i$). This extra condition does not hold for $\hat{y}_i > y_i$ when $M = 2$, which leads to the existence of $p^A = 0$ underpricing equilibria only. When $M > 2$, there exist underpricing equilibria with $p^A > 0$, in which out-of-equilibrium submission of physical settlement requests does not lead agents with long CDS positions to participate in the second stage of the auction. The details are available upon request.

Step 4. Consider now the optimal physical settlement requests of agents with initial short CDS positions. We need consider only those agents who are allowed to hold bonds after the auction. As part of the equilibrium constructed in step 3, these agents receive q_i units of bonds, as given in (A16). So, we can write condition (A13) as

$$\frac{\partial \Pi_i}{\partial y_i} = -\frac{\partial p^A(NOI)}{\partial NOI} \frac{NOI + \sum_{j \in \mathcal{N}_+ : n_j < 0} (n_j - y_j)}{M} - \frac{v - p^A(NOI)}{M}. \tag{A18}$$

For simplicity, we solve for the interior solution so that $\frac{\partial \Pi_i}{\partial y_i} = 0$. Direct computations show that in such an equilibrium it must be the case that

$$NOI + \sum_{j \in \mathcal{N}_+ : n_j < 0} (n_j - y_j) = \left(v - p^A(NOI)\right) / \left| \frac{\partial p^A(NOI)}{\partial NOI} \right|. \tag{A19}$$

Now consider the optimal physical settlement requests of agents with initial long CDS positions. If these agents receive a zero equilibrium bond allocation, conditions (A10) and (A13) imply that their optimal physical settlement requests satisfy

$$y_i = \max\left\{ n_i - \left(v - p^A(NOI)\right) / \left| \frac{\partial p^A(NOI)}{\partial NOI} \right|, 0 \right\}. \tag{A20}$$

Using equilibrium condition (A19) together with condition (A17), we can see that agents with initial long CDS positions will receive a zero equilibrium bond allocation at the second stage if

$$n_i \geq \frac{\left(v - p^A(NOI)\right) / \left| \frac{\partial p^A(NOI)}{\partial NOI} \right|}{M - 1}. \tag{A21}$$

Assumption (A7), along with condition (9), guarantee an interior solution for the optimal physical settlement requests of agents with initial long CDS positions.

Step 5. Finally, the optimal physical requests of the agents must sum to the NOI:

$$\sum_{i : n_i > 0} \left(n_i - \frac{v - p^A(NOI)}{\left| \frac{\partial p^A(NOI)}{\partial NOI} \right|} \right) + \sum_{i \in \mathcal{N}_+ : n_i < 0} y_i = NOI. \tag{A22}$$

Using (A19), we can write (A22) as

$$\sum_{i:n_i>0} n_i + \sum_{i\in\mathcal{N}_+:n_i<0} n_i - \frac{v - p^A(NOI)}{\left|\frac{\partial p^A(NOI)}{\partial NOI}\right|}(K+1) = 0, \tag{A23}$$

where K is the number of agents with initial long CDS positions. Consider the case where $p^A(NOI) = v - \delta \times NOI$. Under this specification,

$$\frac{v - p^A(NOI)}{\left|\frac{\partial p^A(NOI)}{\partial NOI}\right|} = NOI.$$

Condition (A22) gives a simple formula for the NOI:

$$NOI = \frac{\sum_{i:n_i>0} n_i + \sum_{i\in\mathcal{N}_+:n_i<0} n_i}{K+1} > 0. \tag{A24}$$

QED.

Proof of Proposition 5

As usual, we focus on the case where $NOI > 0$. Note that the pro-rata allocation rule satisfies the *majority property* (Kremer and Nyborg, 2004b): an agent whose demand at the clearing price is above 50% of the total demand is guaranteed to be allocated at least $(50\% + \eta) \times NOI$, where $\eta > 0$.

First, suppose that $v \leq IMM + s$. The proof that p^A cannot be above v is the same as in Proposition 1. We now prove that p^A cannot be below v. Suppose instead that $p^A < v$. The part of agent i's utility that depends on her equilibrium allocation and the final price is

$$(v - p^A) \times q_i - p^A \times (n_i - y_i).$$

Suppose first that there is at least one agent for which $q_i < 0.5$. Suppose that this agent changes her demand schedule to

$$x_i'(p) = \begin{cases} NOI, & p \leq p^A + \varepsilon \\ 0, & \text{otherwise}, \end{cases} \tag{A25}$$

where $0 < \varepsilon < v - p^A$. After this deviation, the new clearing price is $p^A + \varepsilon$. Since

$X_{-i}(p^A + \varepsilon) < NOI$ (otherwise $p^A + \varepsilon$ would have been the clearing price), agent i demands more than 50% at $p^A + \varepsilon$, and under the pro-rata allocation rule receives $q_i' > 0.5 \times NOI$. The lower bound on the relevant part of agent i's utility is now

$$(v - p^A - \varepsilon) \times 0.5 \times NOI - (p^A + \varepsilon) \times (n_i - y_i).$$

We can write the difference between agent i's utility under deviation and her utility under the assumed equilibrium as follows:

$$(0.5 \times NOI - q_i) \times (v - p^A) - \varepsilon(n_i - y_i + 0.5 \times NOI). \tag{A26}$$

For small enough ε and under the assumption that $p^A < v$, (A26) is greater than zero, so equilibria with $p^A < v$ cannot exist.

If there are no agents with $q_i < 0.5 \times NOI$, we have an auction that has only two bidders. In this case, each bidder gets exactly $0.5 \times NOI$. At price $p^A + \varepsilon$ ($0 < \varepsilon < IMM + s - v$), there is at least one player (player i), for which $x_i(p^A + \varepsilon) < 0.5 \times NOI$. Then, if the opposite agent uses demand schedule (A25), the new clearing price will be $p^A + \varepsilon$ and this agent will receive at least $(0.5 + \eta) \times NOI$. For small enough ε the difference between agent i's utility under the deviation and her utility under the assumed equilibrium is

$$\eta \times (v - p^A) - \varepsilon(n_i - y_i + (0.5 + \eta) \times NOI) > 0. \tag{A27}$$

Therefore, equilibria with $p^A < v$ cannot exist. We conclude that if $v \leq IMM + s$, $p^A = v$ is the only clearing price in any equilibrium under the pro-rata allocation rule.

Finally, suppose that $IMM + s < v$. The proof for this case is the same, except that there is no feasible deviation to a higher price if $p^A = IMM + s$. Hence, $p^A = IMM + s < v$ is the only clearing price in any equilibrium under the pro-rata allocation rule. QED.

Tables and Figures

Table 1: *Washington Mutual Market Quotes*

Dealer	Bid	Offer
Banc of America Securities LLC	62.5	64.5
Barclays Bank PLC	62	64
BNP Paribas	63	65
Citigroup Global Markets Inc.	62.25	64.25
Credit Suisse International	61.125	63.125
Deutsche Bank AG	62	64
Dresdner Bank AG	64.25	66.25
Goldman Sachs & Co.	62.25	64.25
HSBC Bank USA, National Association	63	65
J.P. Morgan Securities Inc.	63	65
Merrill Lynch, Pierce, Fenner & Smith Incorporated	63	65
Morgan Stanley & Co. Incorporated	62.25	64.25
The Royal Bank of Scotland PLC	63.5	65.5
UBS Securities LLC	62.25	64.25

Table 1 shows the two-way quotes submitted by dealers at the first stage of the Washington Mutual auction.

Table 2: *Auction Summaries*

Name	Date	Net Notional, $NETCDS$	Initial Market Midpoint, IMM	Net Open Interest, NOI	Final Price
Dura	28 Nov 2006	NA	24.875	20	24.125
Dura Subordinated	28 Nov 2006	NA	4.250	77	3.500
Quebecor	19 Feb 2008	NA	42.125	66	41.250
Lehman Brothers	10 Oct 2008	5,568	9.750	4,920	8.625
Washington Mutual	23 Oct 2008	2,946	63.625	988	57.000
Tribune	6 Jan 2009	1,231	3.500	765	1.500
Lyondell	3 Feb 2009	773	23.250	143	15.500
Nortel Corp.	10 Feb 2009	520	12.125	290	12.000
Smurfit-Stone	19 Feb 2009	362	7.875	128	8.875
Chemtura	14 Apr 2009	498	20.875	98	15.000
Great Lakes‡	14 Apr 2009	241	22.875	130	18.250
Rouse†	15 Apr 2009	NA	28.250	8	29.250
Abitibi†	17 Apr 2009	428	3.750	234	3.250
Charter Comm	21 Apr 2009	NA	1.375	49	2.375
Capmark†	22 Apr 2009	NA	22.375	115	23.375
Idearc	23 Apr 2009	1,167	1.375	889	1.750
Bowater	12 May 2009	426	14.000	117	15.000
R.H.Donnelly Corp.	11 Jun 2009	1,797	4.875	143	4.875
General Motors	12 Jun 2009	2,360	11.000	-529	12.500
Visteon	23 Jun 2009	532	4.750	179	3.000
Six Flags	9 Jul 2009	257	13.000	-62	14.000
Lear	21 Jul 2009	628	40.125	172	38.500
CIT	1 Nov 2009	3,078	70.250	728	68.125
Dynegy	29 Nov 2011	662	69.500	-61	71.250
PMI Group	13 Dec 2011	1,750	18.125	375	16.500
AMR Corp	15 Dec 2011	338	22.000	-119	23.500

Table 2 summarizes the auction results for 26 US firms for which TRACE data are available. It reports the settlement date, net CDS notional values (in millions of USD), initial market midpoint (per 100 of par), net open interest (in millions of USD), and final auction settlement price (per 100 of par). All credit events are Chapter 11 with the exception of the ones denoted by † (failure to pay) and ‡ (Chapter 11 of Chemtura).

Table 3: *Tradable Deliverable Bond Summary Statistics*

Name	Number of deliverable bonds	Notional amount of bonds outstanding, NAB	NOI/NAB (%)	Average price on the day before the auction
Dura	1	350	5.71	25.16
Dura Subordinated	1	458	16.79	5.34
Quebecor	2	600	11.00	42.00
Lehman Brothers	157	42,873	11.47	12.98
Washington Mutual	9	4,750	20.80	64.79
Tribune	6	1,346	56.81	4.31
Lyondell	3	475	30.15	26.57
Nortel Corp.	5	3,150	9.22	14.19
Smurfit-Stone	5	2,275	5.65	7.77
Chemtura	3	1,050	9.40	26.5
Great Lakes	1	400	32.65	26.71
Rouse	4	1,350	0.63	29.00
Abitibi	10	3,000	7.81	4.61
Charter Communications	17	12,769	0.38	2.00
Capmark	2	1,700	6.79	22.75
Idearc	1	2,850	31.21	2.15
Bowater	6	1,875	6.27	14.12
R.H.Donnelly Corp.	7	3,770	3.81	5.12
General Motors	16	18,180	-2.91	11.17
Visteon	2	1,150	15.62	4.87
Six Flags	4	1,495	-4.14	13.26
Lear	3	1,299	13.28	39.27
CIT	281	22,585	3.29	69.35
Dynegy	7	3,782	-1.62	69.82
PMI Group	3	661	56.82	21.62
AMR Corp	13	2,062	-5.79	21.85

Table 3 provides summary statistics of deliverable bonds for 26 US firms for which TRACE data are available. Column three reports the ratio of net open interest (NOI) from Table 2 to the notional amount outstanding of deliverable bonds (in millions of USD). The last column shows a weighted average bond price on the day before the auction, constructed as described in Section 4.1.

Table 4: *Mispricing and the NOI*

	NOI	NOI/N	$NOI/NETCDS$	NOI/NAB
	OLS Regression			
α	0.90	0.93	0.98	0.99
	(20.58)	(21.59)	(17.20)	(25.29)
β	-0.07	-3.32	-0.41	-0.91
	(-1.65)	(-2.77)	(-3.07)	(-4.85)
$R^2(\%)$	10.20	24.26	33.12	49.51
	Median Regression			
α	0.96	1.01	0.99	1.02
	[0.89,1.07]	[0.96,1.13]	[0.90,1.15]	[0.93,1.09]
β	-0.06	-4.57	-0.33	-1.02
	[-0.11,0.06]	[-6.88,-1.76]	[-0.63,-0.06]	[-1.38,-0.53]

Table 4 shows the results of the univariate OLS and median regressions

$$p^A/p^{-1} = \alpha + \beta \times NOI/S + \varepsilon,$$

where S is 1, N, $NETCDS$, or NAB. The 95% confidence bounds in the median regressions are computed using bootstrap.

Figure 1: *IMM Determination: The Case of Washington Mutual*

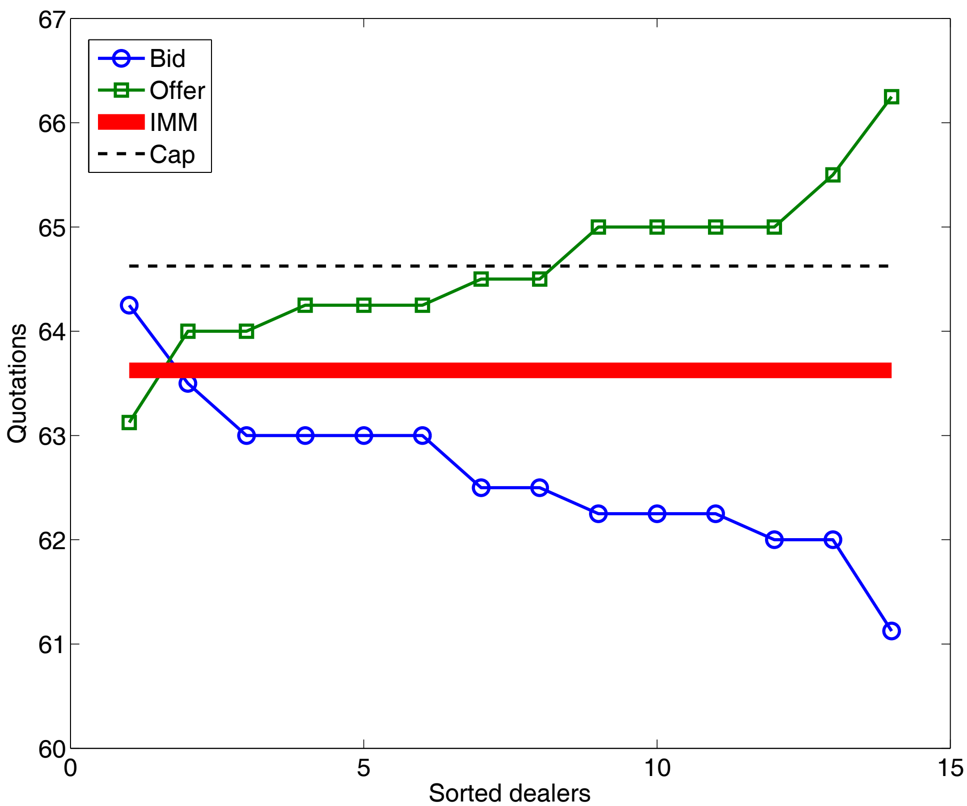


Figure 1 displays all bids (sorted in descending order) and all offers (sorted in ascending order). Tradeable quotes (bid greater than offer) are discarded for the purposes of computing IMM. Dealers quoting tradeable markets must pay a penalty (adjustment amount) to ISDA. The cap price is higher than the IMM by 1% of par and is used in determining the final price. (If the open interest is to buy, the cap price is set below the IMM.)

Figure 2: *Trading Volume*

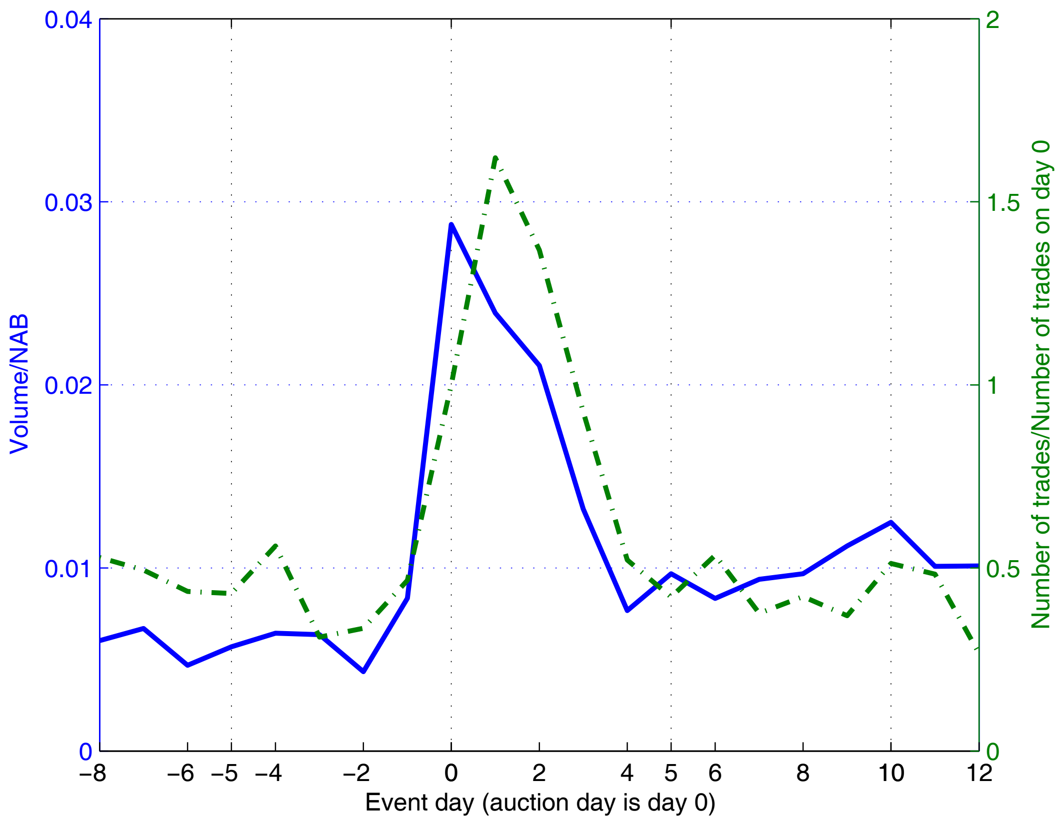
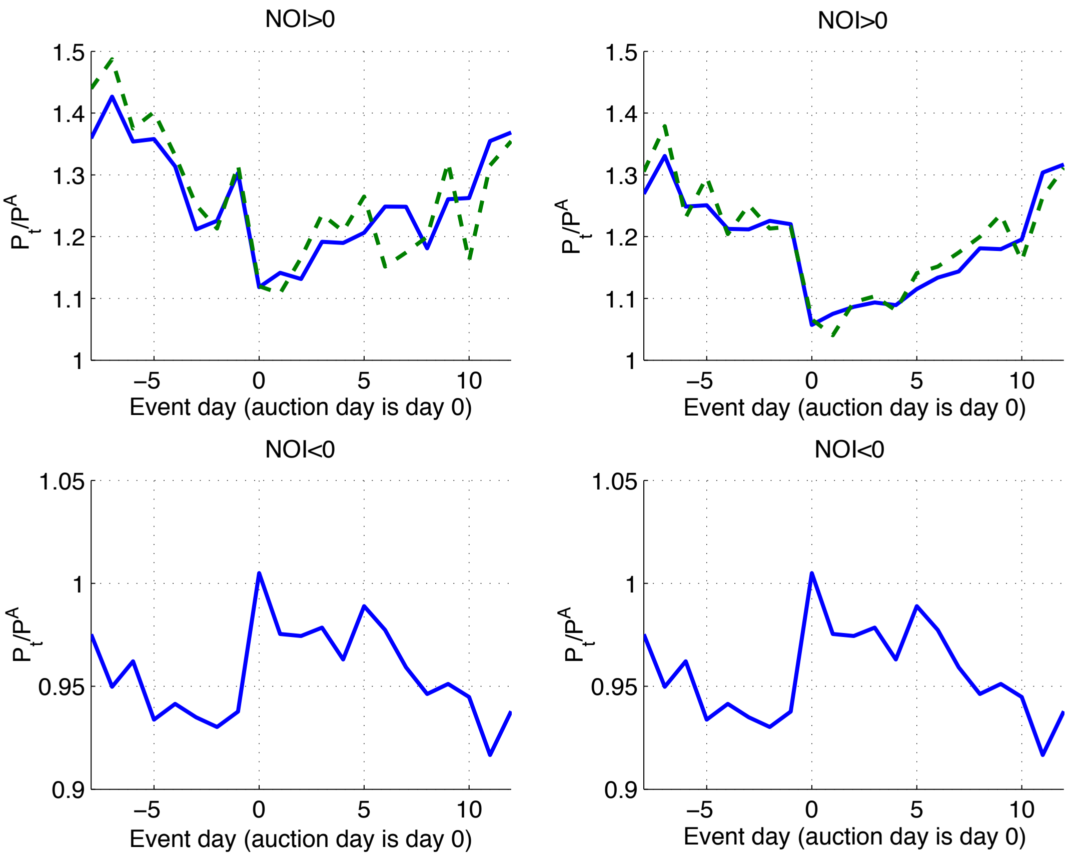


Figure 2 displays the daily trading volume and the number of trades, weighted equally across the 26 auctions reported in Table 2. For each auction, the trading volume (the blue line) is computed as a percentage of NAB; the number of trades (the green line) is normalized by the number of trades on the day of the auction (day 0).

Figure 3: *Price Impact*



(*a*) (*b*)

Panel (a) displays daily bond prices, normalized by the auction final price, p_t/p^A, and weighted equally across the 25 auctions reported in Table 2 (the Charter auction is excluded due to a lack of reliable bond data). Panel (b) shows the same prices but excluding the Tribune auction, which has the largest degree of underpricing. The blue line shows the prices, based on all available bond issues. The green line shows prices that are based only on bond issues with the lowest price.

Figure 4: *Bidding Schedules: The Case of Washington Mutual*

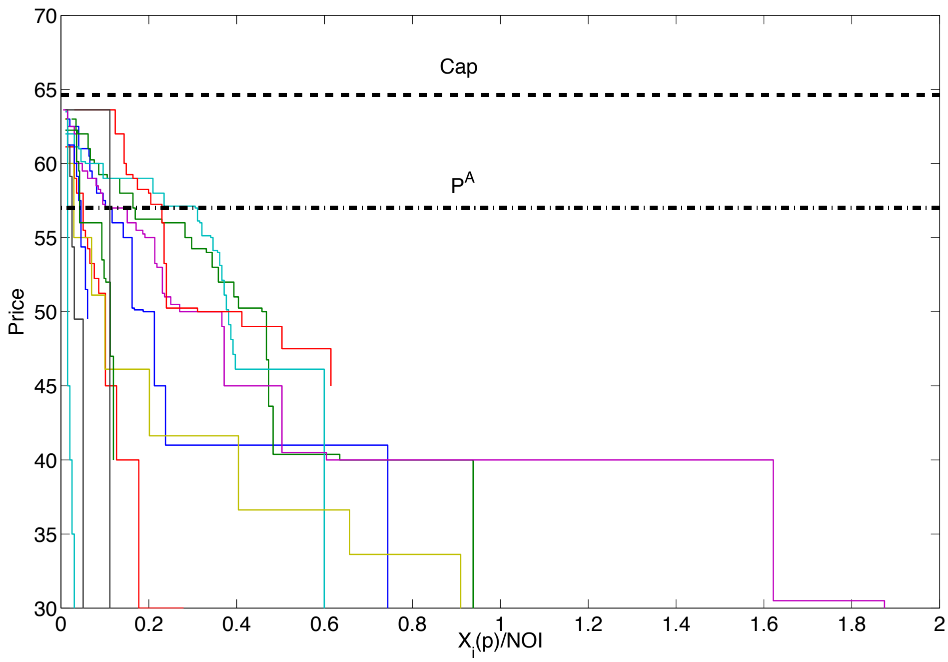


Figure 4 displays all bids submitted at the second stage of the Washington Mutual auction. Each line represents prices and quantities submitted by a dealer.

Figure 5: *Washington Mutual Bond Prices*

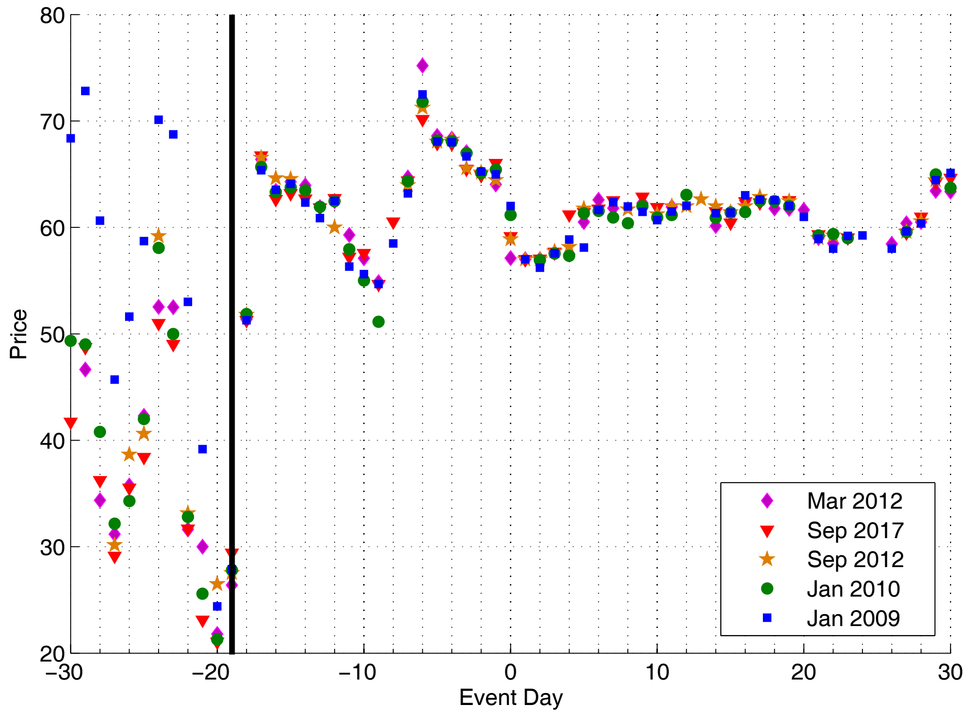


Figure 5 shows daily prices of Washington Mutual outstanding bond issues around the day of bankruptcy (indicated by a vertical black line). The legend shows the maturity date of each issue. The daily price at a given date is a volume-weighted average for all trades at this date. Further details on the construction of this graph are given in Section 4.1.

Figure 6: *Price Discount*

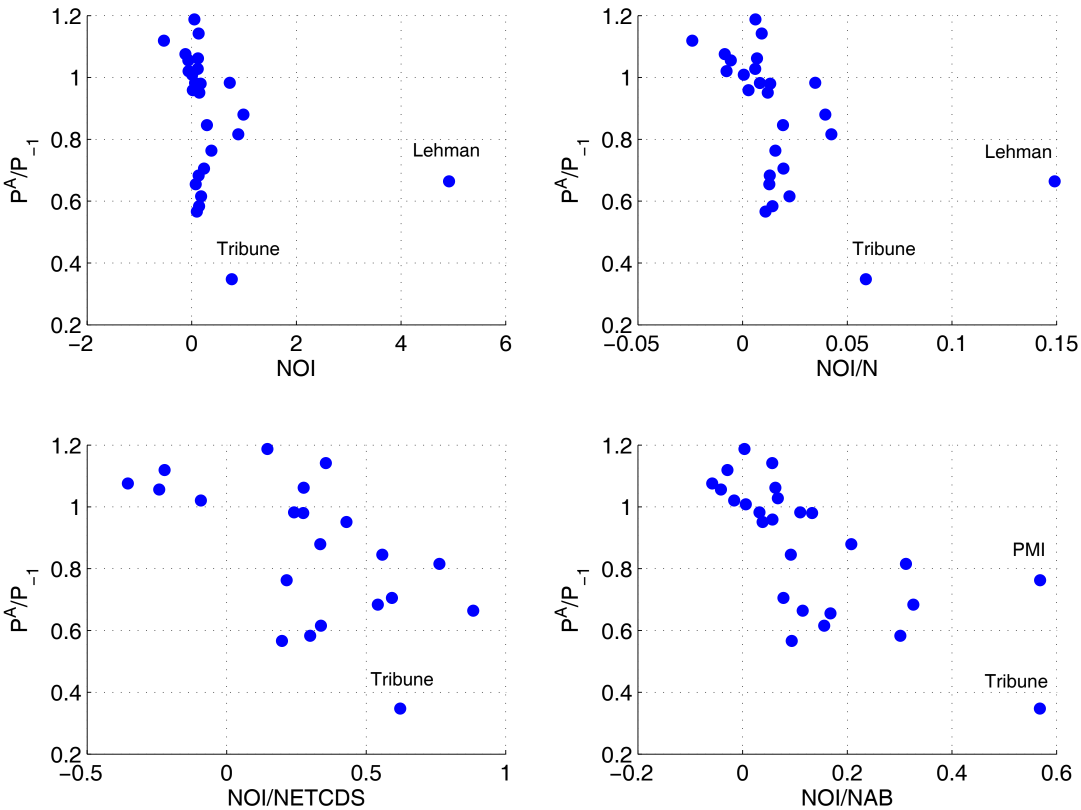


Figure 6 plots the ratio of the final auction price to the weighted-average market price of bonds a day before the auction against the NOI normalized in four different ways: (i) no normalisation (upper left panel); (ii) number of auction participants, N (upper right panel); (iii) net CDS notional value (lower left panel, data for 20 auctions only); (iv) notional amount of deliverable bonds, NAB (lower right panel). We identify extreme observations explicitly.

Figure 7: *Price Discount Over Time*

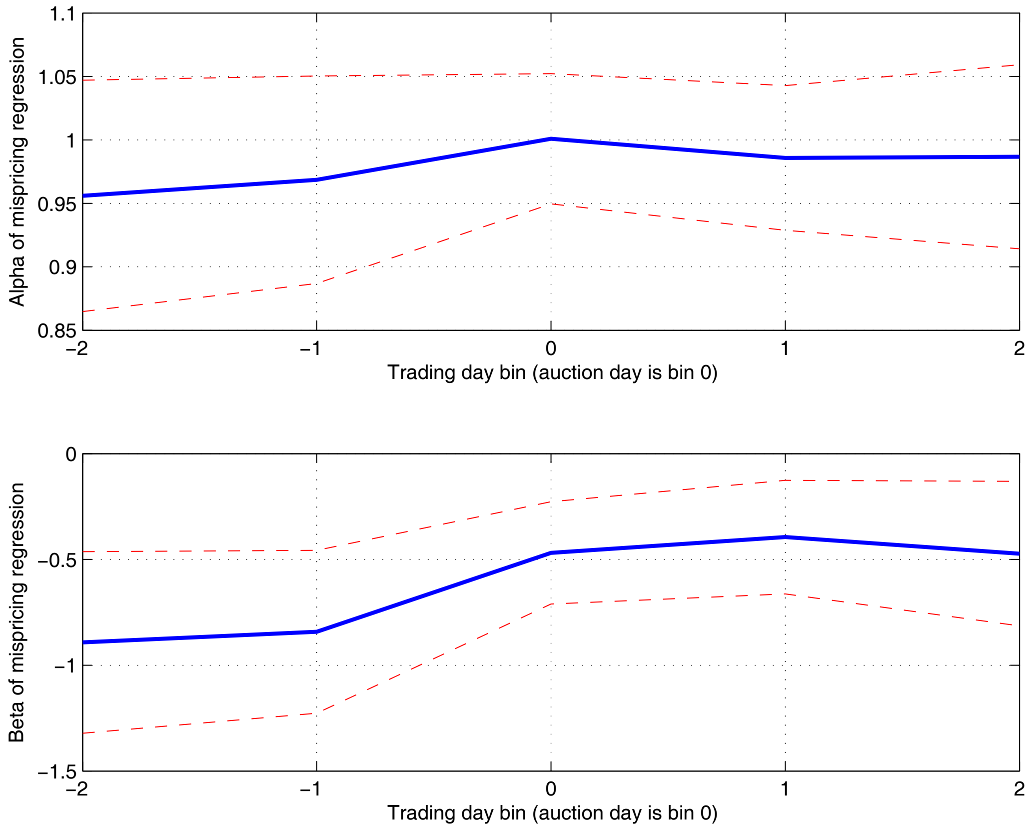


Figure 7 shows time series of α and β together with the respective 95% confidence bounds from the regression

$$p^A/p_t = \alpha + \beta \times NOI/NAB + \varepsilon$$

implemented for each t from the event window [-8, +12]. Because of low liquidity, we construct the reference price p_t by binning trading days together. Bin -2 combines days from -8 to -4; bin -1 combines days from -3 to -1; bin 0 is day 0; bin 1 combines days from 1 to 3; bin 2 combines days from 4 to 8.

Figure 8: *Bids at the Cap Price*

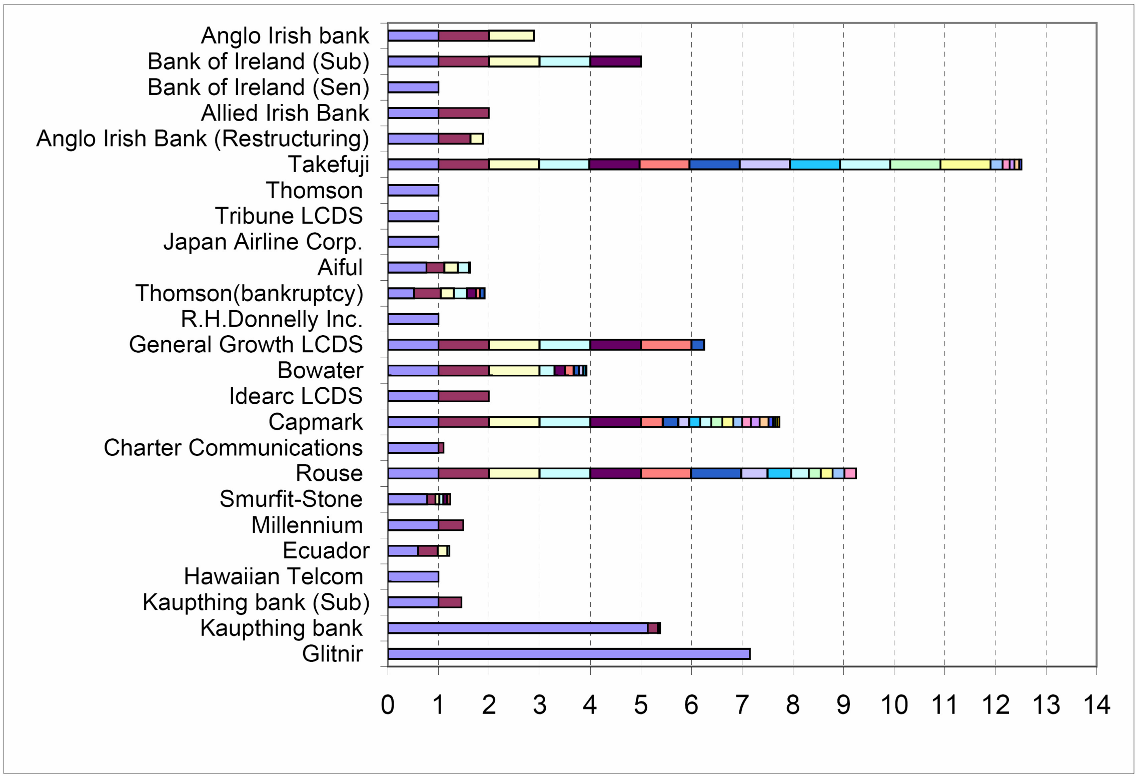


Figure 8 shows individual bids scaled by the NOI at the cap price (in auctions where the price is capped). Each bid within an auction is represented by a different colour.